LESSON 4.4

Applying Matrix Multiplication

CAREER SPOTLIGHT: Bioinformatics Scientists

Occupation Description

Bioinformatics scientists conduct research using bioinformatics theory and methods in areas such as pharmaceuticals, medical technology, biotechnology, computational biology, proteomics, computer information science, biology and medical informatics. They may design databases and develop algorithms for processing and analyzing genomic information, or other biological information.

Education

Bioinformatics scientists typically need a bachelor's degree for entry into the occupation.

Potential Employers

The largest employers of biological scientists are as follows:

R & D in the physical, engineering, and life sciences	23%
Educational services; state, local, and private	13%
Healthcare and social assistance	5%
Manufacturing	3%

Watch a video about bioinformatics scientists: https://cdn.careeronestop.org/OccVids/OccupationVideos/19-1020.01.mp4

Career Cluster

Science, Technology, Engineering & Mathematics

Career Pathway

Science and Mathematics

Career Outlook

- Salary Projections:
  Low-End Salary, $49,060
  Median Salary, $85,290
  High-End Salary, $137,030
- Jobs in 2018: 44,700
- Job Projections for 2028: 45,700 (increase of 2%)

Algebra II Concepts

- Understand how bioinformatics scientists use matrix multiplication to process data.

Is this a good career for me?

Bioinformatics scientists typically do the following: Develop software or applications for scientific or technical use.

- Prepare scientific or technical reports or presentations.
- Analyze biological samples.
- Review professional literature to maintain professional knowledge.
- Develop technical or scientific databases.
- Research genetic characteristics.

Lesson Objective

In this lesson, you will understand how bioinformatics scientists use matrix multiplication to process data.

Multiplying Matrices

A **matrix** is a rectangular array of values, or **elements**, arranged into rows and columns. It's size is described using its **dimensions**, which are always given as rows x columns. For example, the matrix below is a 2 x 3 matrix.

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}$$

The multiplication of matrices is always done by multiplying a row on the left by a column on the right. This row and column must have the same number of elements in order to be multiplied. When multiplying a row by a column, add the partial products made by multiplying the corresponding elements from the row and column matrix. The product of a row times a column is a single element.

$$\begin{bmatrix} 4 & -3 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -8 \\ 5 \end{bmatrix} = \left[(4 \times 3) + \left(-3 \times (-8)\right) + (-1 \times 5)\right] = [12 + 24 + (-5)] = [31]$$

When multiplying larger matrices, each row of the left-hand matrix is multiplied by each column of the right-hand matrix. This means matrices may only be multiplied when the number of columns of the left-hand matrix is equal to the number of rows of the right-hand matrix. The easiest way to check this is to write the dimensions of the matrices next to each other and ensure the middle two dimensions are equal. The outer dimensions are the dimensions of the product matrix.

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix}$$

The dimensions here are (2 x 3)(3 x 2). This means the matrices may be multiplied and the product matrix will be 2 x 2. The row and column being multiplied together define the product element's position in the product matrix.

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0(9) + (-5)(7) + (-9)(0) & \\ & \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -35 & 0(5) + (-5)(4) + (-9)(-2) \\ & \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -35 & -2 \\ (-1)(-9) + 0(7) + 3(0) & \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -35 & -2 \\ 9 & -1(5) + 0(4) + 3(-2) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -9 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} -9 & 5 \\ 7 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -35 & -2 \\ 9 & -11 \end{bmatrix}$$

1 Step Into the Career: Multiplying 2x2 Matrices

Bella is analyzing the probability of several genetic traits as part of her career as a bioinformatic scientist. In one study, she constructs a probability matrix for the distribution of hair color and texture using the information on the following table:

	Blond	Brown
Straight	0.2	0.3
Curly	0.1	0.4

In the population she is studying, there is a probability that mutations may occur that will affect this matrix by multiplying the probability matrix from the table above by the matrix below:

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

Determine the probability of each combination of genetic traits if the mutation occurs in the population.

Devise a Plan

Step 1: Construct a matrix multiplication problem to describe the situation.

Step 2: Multiply the matrices.

Step 3: Interpret the results of the product matrix.

Walk Through the Solution

Step 1: Diana begins by writing the 2 x 2 matrix describing the distribution of the hair color and texture.

$$\begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix}$$

Next, she writes the matrix representing the probability of genetic mutation ***on the left*** of this matrix. This is important because, unlike multiplication of numbers, **matrix multiplication is NOT commutative – the order matters!**

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}\begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix}$$

Step 2: Bella begins by ensuring that the matrices may be multiplied and what the dimensions of the product matrix will be if multiplication is possible.

(2 x 2)(2 x 2) → (2 x 2)

This means the multiplication is possible and the product matrix will also be 2 x 2. Many applications involve **square matrices**, which are matrices with the same number of rows and columns. In general, square dimensions of the same size may always be multiplied and always produce a product matrix of the same dimensions.

Bella then begins by multiplying the first row on the left by the first column on the right.

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9(0.2) + 0.1(0.1) & \\ & \end{bmatrix} = \begin{bmatrix} 0.19 & \\ & \end{bmatrix}$$

Next, she multiplies the first row on the left by the second column on the right.

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9(0.2) + 0.1(0.1) & 0.9(0.3) + 0.1(0.4) \\ & \end{bmatrix} = \begin{bmatrix} 0.19 & 0.31 \\ & \end{bmatrix}$$

Bella then repeats this process by multiplying the second row on the left through the two columns on the right.

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9(0.2) + 0.1(0.1) & 0.9(0.3) + 0.1(0.4) \\ 0.1(0.2) + 0.9(0.1) & \end{bmatrix} = \begin{bmatrix} 0.19 & 0.31 \\ 0.11 & \end{bmatrix}$$

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.9(0.2) + 0.1(0.1) & 0.9(0.3) + 0.1(0.4) \\ 0.1(0.2) + 0.9(0.1) & 0.1(0.3) + 0.9(0.4) \end{bmatrix} = \begin{bmatrix} 0.19 & 0.31 \\ 0.11 & 0.39 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.19 & 0.31 \\ 0.11 & 0.39 \end{bmatrix}$$

Step 3: To interpret the results of the product matrix, Bella places the labels describing the genetic traits back on the matrix in a table.

	Blond	Brown
Straight	0.19	0.31
Curly	0.11	0.39

The mutation causes some minor changes in the distribution of the genetic traits in this population.

On the Job: Apply Multiplying 2x2 Matrices

1. As a bioinformatic scientist, Hector is studying the genetic traits of eye colors and shape in a population. He constructs a table showing the probability of these traits occurring.

	Brown	Blue
Round	0.25	0.15
Almond-shaped	0.35	0.25

It is proposed that a genetic mutation has occurred in the population that will multiply this matrix from the table by the matrix below:

$$\begin{bmatrix} 0.15 & 0.85 \\ 0.85 & 0.15 \end{bmatrix}$$

Determine the probability of the genetic traits Hector is studying if these mutations occur.

2 Step Into the Career: Multiplying Matrices

Sahale was analyzing the connection between genetic diseases and different ethnic groups in his job as a bioinformatic scientist. In order to do this, he uses an **incidence matrix**. This incidence matrix uses a 1 when there is a connection between the value of the row and column and a 0 when there is not. The table below shows his findings:

		Disease			
		A	**B**	**C**	**D**
Ethnic Group	**W**	0	1	1	0
	X	0	1	0	1
	Y	0	1	1	1
	Z	1	0	0	0

By squaring this matrix, Sahale can make predictions about offspring from mixed ethnic groups contracting the genetic diseases. What is the square of this incidence matrix?

Devise a Plan

Step 1: Write the matrix multiplication to find the square of the incidence matrix.

Step 2: Perform the matrix multiplication.

Step 3: State the results of the matrix multiplication in the context of the problem.

Walk Through the Solution

Step 1: Sahale constructs the following 4 x 4 matrix from the data.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Step 2: To square the matrix, Sahale makes a copy of the incidence matrix on the left.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The multiplication of the matrices is then started by multiplying the first row of the left matrix by the first column of the right matrix: $0 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

This is repeated as the first row is multiplied by the next three columns:

$$0 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2$$

$$0 \cdot 1 + 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 = 1$$

$$0 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 2 \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

This process is then repeated as the second row is multiplied through the four columns.

$$0 \cdot 0 + 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 = 1$$

$$0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 0 = 1$$

$$0 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 = 0$$

$$0 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 0 = 1$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ & & & \\ & & & \end{bmatrix}$$

The third and fourth rows are then multiplied through the four columns in a similar manner. The final result is the square of the incidence matrix:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Step 3: Sahale then places the square of the incidence matrix into the original table.

		Disease			
		A	**B**	**C**	**D**
Ethnic Group	**W**	0	2	1	2
	X	1	1	0	1
	Y	1	2	1	2
	Z	0	1	1	0

In this application, if the child is assigned the ethnic group of the mother, the value shows the relative chance that these children could be susceptible to a genetic disease if the father is from a different ethnic group. In general, the higher the number, the more possible combinations that could possibly carry this genetic disease, so a 2 indicates a higher likelihood that the disease would occur in this combination.

On the Job: Applying Multiplying Matrices

2. Angelique is a bioinformatic scientist and attempting to trace genetic traits in different ethnic groups for a study. Her data provides the following incidence matrix.

		Trait			
		A	**B**	**C**	**D**
Ethnic Group	**W**	0	0	1	0
	X	1	1	0	1
	Y	1	0	0	0
	Z	0	1	0	1

She wants to construct the square of the incidence matrix to investigate how traits may be passed on to children of mixed ethnic groups. What is the square of this incidence matrix?

Career Spotlight: Practice

3. Valentino is working as a bioinformatic scientist and needs to analyze how possible mutations could affect genetic traits in a population. To do this, he must calculate a rating found by multiplying the following matrices. What is the product matrix?

$$\begin{bmatrix} 0.9 & 1.1 & 0.8 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$$

4. Aundria is analyzing how genetic mutations may affect the distribution of a genetic traits in a certain population. To determine this, she must multiply the following matrices. What is the product matrix?

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.3 & 0.1 \\ 0.4 & 0.2 \end{bmatrix}$$

5. As part of her job as a bioinformatic scientist, Mary is analyzing connections between genetic traits and ethnic groups. Her data finds the following incidence matrix showing which traits are prevalent within which ethnic groups.

		Trait			
		A	**B**	**C**	**D**
Ethnic Group	**W**	1	0	0	1
	X	1	0	1	1
	Y	0	1	0	1
	Z	0	1	1	0

To investigate possible connections in offspring with mixed ethnic heritage, she decides to square the incidence matrix. What is this matrix?

Devise a Plan

Step 1: Write the incidence matrix.

Step 2: ____?____

Step 3: ____?____

Career Spotlight: Check

6. Kavik needs to multiply matrices as part of his job as a bioinformatic scientist. Which of the following pairs of matrices may be multiplied? Select all that apply.

a. $\begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 7 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} -6 & 5 \\ 4 & -4 \end{bmatrix}$

c. $\begin{bmatrix} -3 & 6 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$

d. $\begin{bmatrix} 0 \\ 9 \\ -5 \end{bmatrix}\begin{bmatrix} -5 & -5 \end{bmatrix}$

e. $\begin{bmatrix} 8 & -9 & 6 \\ 6 & 6 & 3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 4 & 5 \end{bmatrix}$

f. $\begin{bmatrix} -8 \\ 0 \\ -4 \end{bmatrix}\begin{bmatrix} -4 & 4 & 8 \end{bmatrix}$

7. Fernanda is investigating the possible changes in genetic traits in a population in her job as a bioinformatic scientist. What is the product matrix of the matrix multiplication that results from her data?

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}\begin{bmatrix} 0.25 & 0.35 \\ 0.15 & 0.25 \end{bmatrix}$$

a. $\begin{bmatrix} 0.36 & 0.54 \\ 0.04 & 0.06 \end{bmatrix}$

b. $\begin{bmatrix} 0.225 & 0.035 \\ 0.015 & 0.225 \end{bmatrix}$

c. $\begin{bmatrix} 0.16 & 0.26 \\ 0.24 & 0.34 \end{bmatrix}$

d. $\begin{bmatrix} 0.24 & 0.34 \\ 0.16 & 0.26 \end{bmatrix}$

8. Lucas is analyzing the incidence of genetic traits in different ethnic groups and obtains the following incidence matrix.

		Trait			
		A	B	C	D
Ethnic Group	W	0	1	1	0
	X	0	1	1	1
	Y	0	0	1	0
	Z	0	1	0	1

Which of the following is the square of this incidence matrix?

a. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$

9. Mario wants to obtain a score for predicting the genetic trait in a certain population. To do so, he must multiply the following matrices:

$$\begin{bmatrix} 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 6 \end{bmatrix}$$

What is the result of this multiplication?

a. $[34]$ **b.** $[29]$
c. $[46]$ **d.** $[47]$

10. Ayiana wants to analyze how genetic mutation could affect a certain population by multiplying the following matrices:

$$\begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix} \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

Her intern decides to attempt to find the results this way:

$$\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

Explain the intern's error and show why this is incorrect.