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About Pathway2Careers

As NS4ed's flagship program, Pathway2Careers (P2C) is a career readiness solution that support schools in their efforts to prepare students for high-value careers in their communities. P2C delivers access to labor market data to encourage the use of data-informed practices in career education through a number of services, including labor market data tools, career-focused curricula, Perkins V support, and more. As a comprehensive career readiness solution, P2C serves administrators, faculty, students, and parents in making informed decisions that promote employment success.

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LESSON 1.1

Real Numbers

CAREER PREPARATION: Essential Number Sense Skills

Did you know?

Physicists use different types of numbers to examine and understand physical phenomena.

Consider this situation…

A physicist makes a calculation to determine how long it will take a ball to fall 32 feet. Her calculation shows that the time it takes for the ball to fall is $\sqrt{2}$, or about 1.4142, seconds. Explain why $\sqrt{2}$ is not a rational number.

To determine the time it takes, the physicist solves the equation $0 = -16t^2 + 32$. In solving this equation, the physicist gets $t = \sqrt{2}$.

A *rational number* is any number that can be written as ratio of two integers. It can be shown that $\sqrt{2}$ is not a ratio of two integers.

Let's find out… why $\sqrt{2}$ is not a rational number so you can classify rational and irrational numbers on your own in the **Career Preparation Exercises.**

Lesson Objective

In this lesson, you will understand properties of real numbers.

- You will understand that rational numbers and irrational numbers make up real numbers.
- You will understand that the sums and products of rational numbers are rational numbers.

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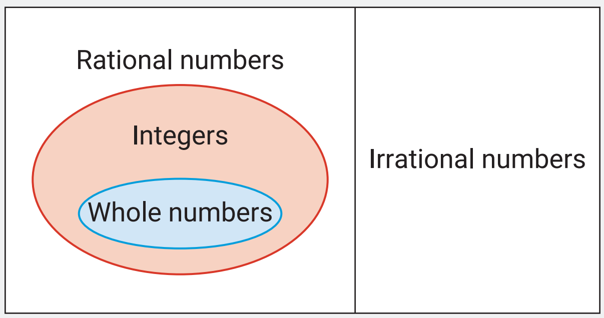
Number Sense Essentials

Recall that **rational numbers** are all numbers that can be written in the form of a ratio $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Rational numbers are part of the *real numbers*. **Real numbers** are all numbers that can be represented on a number line. Any real number that is not a rational number is an *irrational number*. **Irrational numbers** are all numbers that cannot be written as a ratio of integers. The decimal form of an irrational is both non-terminating and non-repeating.

For example, the numbers $\sqrt{2}$ and π are irrational numbers. Their decimal approximations 1.21421… and 3.14159… are non-terminating and non-repeating.

The diagram shows how real numbers can be classified as whole numbers, integers, rational numbers, and irrational numbers.

Real Numbers



Classifying Real Numbers

Example 1 Classifying Real Numbers

Determine whether the real number is rational or irrational. Explain your answer.

a. 5 **b.** $\sqrt{11}$ **c.** $-\frac{21}{4}$ **d.** 2.4

Solution

a. The number 5 can be written as the ratio $\frac{5}{1}$, so it is a rational number.

b. The number $\sqrt{11}$ cannot be written as a ratio of integers, so it is an irrational number.

c. The number $-\frac{21}{4}$ can be written as the ratio $\frac{-21}{4}$, so it is a rational number.

d. The number 2.4 can be written as the ratio $\frac{12}{5}$, so it is a rational number.

Example 2 Determining an Irrational Number

Show why $\sqrt{2}$ is not a rational number.

Solution

Assume that $\sqrt{2}$ is a rational number, so it can be written as the ratio of two integers p and q.

$$\frac{p}{q} = \sqrt{2}$$

You can also assume that the integers p and q are both positive and chosen so that $\frac{p}{q}$ is in simplest form. Then, square $\frac{p}{q}$ and $\sqrt{2}$, so you get the following.

$$\frac{p^2}{q^2} = 2$$

So, p^2 is divisible by 2, which means 2 must be a factor of p because 2 is a prime number. This means p^2 is divisible by 4. This implies that 2 must be a factor q^2 since the quotient of p^2 and q^2 is 2. This means 2 is a factor of q.

Because 2 is factor q and 2 is a factor of p, this tells you that $\frac{p}{q}$ is not in simplest form. This is a contradiction. In math, if you make an assumption that leads to a contradiction, the assumption cannot be true. Therefore, the assumption that $\sqrt{2}$ is a rational number is not true, so $\sqrt{2}$ must be an irrational number.

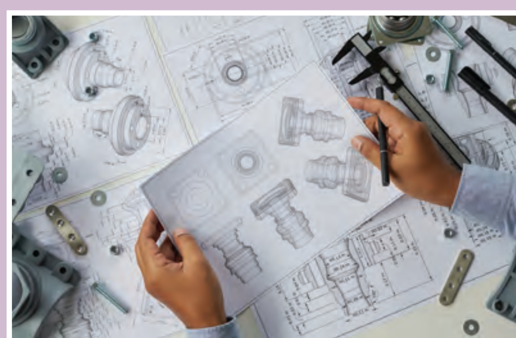
Build Your Skills: Classifying Real Numbers

Determine whether the real number is rational or irrational. Explain your answer.

1. $-\sqrt{3}$
2. $0.\overline{6}$
3. $-\sqrt{\frac{25}{9}}$
4. 3.2
5. Show why $\sqrt{5}$ is not a rational number.

Did you know?

Mechanical drafters make designs that involve calculations with irrational numbers such π and $\sqrt{2}$.



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Number Sense Essentials

The set of real numbers is closed, or has *closure,* under addition, subtraction, and multiplication. **Closure** of a set of numbers for a given operation means that the operation of any two numbers in the set results in another number in the set. For example, the sum of two real numbers is another real number, so the set of real numbers is closed under addition. When accounting for closure of an operation, you must consider all numbers in the set. For example, the set of real numbers is not closed under division because dividing by zero does not result in a real number.

The set of rational numbers is also closed under addition, subtraction, and multiplication, and the set of nonzero rational numbers is closed under division. Irrational numbers are not closed under addition, subtraction, and multiplication.

The following properties are true:

1. The sum and difference of two rational numbers are rational numbers.
2. The product of two rational numbers is a rational number.
3. The sum of an irrational number and a rational number is an irrational number.
4. The product of an irrational number a rational number is an irrational number.

Exploring Properties of Real Numbers

Example 3 Identifying Rational and Irrational Numbers

Identify whether the number is rational or irrational. Explain your answer.

a. $1.3 + \pi$ **b.** $0.75 - \frac{3}{14}$ **c.** $-5(4.\overline{3})$ **d.** $\frac{\sqrt{2}}{5}$

Solution

a. Because 1.3, which can be written as $\frac{8}{5}$, is a rational number and π is an irrational number, the sum is an irrational number.

b. Because both 0.75, which can be written as $\frac{3}{4}$, and $\frac{3}{14}$ are rational numbers, the difference is a rational number.

c. Because -5, which can be written as $\frac{-5}{1}$, and $4.\overline{3}$, which can be written as $\frac{13}{3}$, are rational numbers, the product is a rational number.

d. The quotient $\frac{\sqrt{2}}{5}$ can be rewritten as the product $\frac{1}{5} \cdot \sqrt{2}$. Because $\frac{1}{5}$ is a rational number and $\sqrt{2}$ is an irrational number, the product is irrational.

Example 4 Adding a Rational Number and an Irrational Number

Show why the sum of the rational number 5 and the irrational number $\sqrt{11}$ is an irrational number.

Solution

Assume that $5 + \sqrt{11}$ is a rational number, so it can be written as the ratio of two integers p and q.

$$\frac{p}{q} = 5 + \sqrt{11}$$

Then, if you subtract 5 from $5 + \sqrt{11}$, it is the same as subtracting 5 from $\frac{p}{q}$. You can subtract 5 by writing it as a fraction with the common denominator q.

$$\frac{p}{q} - 5 = \frac{p}{q} - \frac{5q}{q} = \frac{p - 5q}{q}$$

The numerator $p - 5q$ and the denominator q are integers. This shows that $\sqrt{11}$ can be written as the ratio of two integers, which means $\sqrt{11}$ is a rational number, and that is a contradiction. The assumption that $5 + \sqrt{11}$ is a rational number is not true, so $5 + \sqrt{11}$ must be an irrational number.

Build Your Skills: Exploring Properties of Real Numbers

Identify whether the number is rational or irrational. Explain your answer.

6. $1.\overline{72}\left(\frac{3}{2}\right)$

7. $-\sqrt{2} + 1.41$

8. $\frac{\pi}{3}$

9. $-3.14 - \frac{4}{7}$

Show why the statement is true.

10. The sum of the irrational number $\sqrt{3}$ and the rational number $-\frac{3}{4}$ is an irrational number.

11. The product of the rational number 2.3 and the irrational number π is an irrational number.

Did you know?

Statisticians make calculations with data involving rational and irrational numbers to determine statistical measurements.

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Career Preparation: Practice

Determine whether the real number is rational or irrational. Explain your answer.

1. $0.\overline{428571}$

2. $\frac{5}{8}$

3. $\frac{\pi}{2}$

4. $\sqrt{17}$

5. 3.14159

6. $-\sqrt{\frac{64}{121}}$

7. $-\frac{11}{12}$

8. 2π

9. $\sqrt{\frac{2}{49}}$

10. Show why $\sqrt{7}$ is not a rational number.

11. Show why $\sqrt{4}$ is a rational number.

Identify whether the number is rational or irrational. Explain your answer.

12. $\pi(2^2)$

13. $6+\frac{3}{4}$

14. $\frac{3}{4}-\frac{9}{10}$

15. $0.\overline{63}+\frac{1}{3}$

16. $\sqrt{37}+1$

17. $-6.5749+\sqrt{15}$

18. $\left(\sqrt{3}\right)^2$

19. $\frac{1}{5}(3.\overline{5})$

20. $-\left(\sqrt{5^3}\right)$

Show why the statement is true.

21. The product of the rational number $\frac{7}{16}$ and the rational number $\frac{2}{5}$ is a rational number.

22. The sum of the rational number $\frac{3}{4}$ and the rational number $\frac{4}{5}$ is a rational number.

Use It On the Job

23. Renata is a video game designer making an avatar move diagonally across a square room. The length of the diagonal d of a square of side s is $d = s\sqrt{2}$.

Determine a value of s for which d is a rational number. Explain your answer.

24. A police officer is using the formula $s = 2\sqrt{5\ell}$ to estimate the speed s of a car in miles per hour determined by the length ℓ in feet of the skid marks the car makes.

Determine a value of ℓ for which s is a rational number. Explain your answer.

Photo credit: iJeab/Shutterstock.com; Trzykropy/Shutterstock.com

Career Preparation: Check

1. Which number is rational?

A. $\sqrt{5}$

B. π

C. $\pi\sqrt{5}$

D. $\sqrt{9}$

2. Which number is irrational?

A. $\frac{3}{5}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{7}{3}$

D. $\sqrt{144}$

3. Which number is *not* rational?

A. $\sqrt{4}$

B. $\sqrt{\pi} \cdot \sqrt{\pi}$

C. $\sqrt{12} \cdot \sqrt{3}$

D. $2\sqrt{3} - \sqrt{12}$

4. Which number is *not* irrational?

A. $2 + \sqrt{3}$

B. $\sqrt{4} - \sqrt{9}$

C. $\sqrt{4} + \sqrt{3}$

D. $\sqrt{4} \cdot \sqrt{3}$

5. Which statement is *always* true?

A. The difference of two rational numbers is irrational.

B. The difference of two irrational numbers is irrational.

C. The product of an irrational number and a nonzero rational number is irrational.

D. The product of two irrational numbers is irrational.

6. Which statement is *never* true?

A. The sum of a rational number and an irrational number is rational.

B. The sum of two irrational numbers is rational.

C. The product of two irrational is rational

D. The product of a nonzero rational number and an irrational number is rational

7. Identify the rational numbers.

Select all the numbers that apply.

a. $-2(12.\overline{7})$

b. $14.8 + \frac{4}{9}$

c. $\pi\sqrt{81}$

d. $-\left(\sqrt{\pi^2}\right)$

e. $\frac{9}{10} - \frac{1}{3}$

f. $\sqrt{225} - \sqrt{121}$

8. Identify the irrational numbers.
Select all the numbers that apply.

a. $1\frac{9}{16} + 0.75$

b. $-45.\overline{07} + \sqrt{19}$

c. $3\pi + \frac{2}{5}$

d. $0.\overline{157} + \frac{7}{12}$

e. $\frac{\sqrt{3}}{15}$

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9. The table shows properties of real numbers. Identify each property as true or false.

	True	False
The sum and difference of two rational numbers are rational numbers.		
The product of two rational numbers is an irrational number.		
The sum of an irrational number and a rational number is a rational number.		
The product of an irrational number and a nonzero rational number is an irrational number.		
The set of irrational numbers are closed under addition.		

Use It On the Job

10. A ship captain can estimate the distance d in miles to the horizon over water using the formula $d = 1.2246\sqrt{h}$, where h is the height of the captain's eyes above sea level.

For which value of h will d be a rational number?

A. $\sqrt{4}$

B. 9

C. $\sqrt{9}$

D. 14

11. Dyani is an architect in charge of building a sloped roof. She knows the height h of the roof and the length ℓ for it to cover. To find the diagonal length d of the roof, she uses the equation slope equation $d = \sqrt{h^2 + \ell^2}$.

For which values of h and ℓ will d be an irrational number?

A. $h = 3, \ell = 4$

B. $h = 8, \ell = 6$

C. $h = 12, \ell = 5$

D. $h = 15, \ell = 9$

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LESSON 1.1

Real Numbers

Teacher Edition

CAREER PREPARATION: Essential Number Sense Skills

Common Core State Standards

N-RN.1 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Mathematical Practices 1, 2, 3

Lesson Objective

In this lesson, you will understand properties of real numbers.

- You will understand that rational numbers and irrational numbers make up real numbers.
- You will understand that the sums and products of rational numbers are rational numbers.

Vocabulary

- rational numbers
- real numbers
- irrational numbers
- closure

Teaching Support

Number Sense Essentials

Teaching Strategy Students should become familiar with the diagram and understand that if a real number is not rational, it must be irrational. Explain that in later courses, they will learn about other numbers that are not real numbers.

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Classifying Real Numbers

Avoid Common Errors Make sure that students understand that having the square root symbol does not mean that a number is irrational. For example, $\sqrt{4}$ simplifies to 2, a rational number.

Example 1 Classifying Real Numbers

Cooperative Learning Have students work in small groups to write irrational and rational numbers. Then have groups exchange their numbers to check each other's understanding.

Example 2 Determining an Irrational Number

Teaching Strategy Note that this argument can be used for any prime number under the square root symbol.

Build Your Skills: Classifying Real Numbers

Answers

1. irrational; It cannot be written as a ratio of integers.
2. rational; It can be written as the ratio $\frac{2}{3}$.
3. rational; It can be written as the ratio $-\frac{5}{3}$.
4. rational; It can be written as the ratio $\frac{16}{5}$.
5. Assume $\sqrt{5}$ is a rational number written in simplest form $\frac{p}{q}$, where p and q are integers. By squaring, $\frac{p^2}{q^2} = 5$, so p^2 is divisible by 5, which means 5 is also a factor of p because 5 is a prime number. So, p^2 is divisible by 25 and that implies 5 is factor q^2 and a factor of q. This means $\frac{p}{q}$ is not in simplest form, which contradicts the assumption, so $\sqrt{5}$ must be an irrational number.

Number Sense Essentials

Teaching Strategy Closure is an important concept for students to understand as they will later explore the closure of polynomials under addition, subtraction, and multiplication.

Extension Closure can also be determined for subsets. Ask students whether even integers form a closed set under addition, subtraction, and multiplication.

Exploring Properties of Real Numbers

Example 3 Identifying Rational and Irrational Numbers

Teaching Strategy Have students label each number as irrational or rational to help them recognize which property is being applied. For example, for Example 3a, students can write: rational + irrational. This is an example of the third property, so the sum is irrational.

Example 4 Adding a Rational Number and an Irrational Number

Teaching Strategy Point out that since a real number is either rational or irrational, if a number is not rational, it must be irrational.

Teaching Strategy Students can use a similar argument to show that a product of a rational and an irrational number is irrational. Discuss how students can show that by using the operations they can show the sum and product of two rational numbers are rational. Students should be able to extend the argument to show generally that $\frac{a}{b}+\frac{c}{d}$ and $\frac{a}{b}\cdot\frac{c}{d}$ are rational in the Practice exercises.

Build Your Skills: Exploring Properties of Real Numbers

Answers

6. rational; It is the product of two rational numbers.
7. irrational; It is the sum of an irrational number and a rational number.
8. irrational; It can be written as the product of an irrational number and a rational number.
9. rational; It is the difference of two rational numbers.
10. Assume $\sqrt{5}+\left(-\frac{3}{4}\right)$, or $\sqrt{5}-\frac{3}{4}$ is a rational number written in simplest form $\frac{p}{q}$, where p and q are integers. Add $\frac{3}{4}$ to $\sqrt{5}-\frac{3}{4}$ so that means $\sqrt{5}=\frac{p}{q}+\frac{3}{4}=\frac{4p+3q}{4q}$, which is a rational number. This is a contradiction that $\sqrt{5}$ is irrational, so the sum must be irrational.

Career Preparation: Practice

Avoid Common Errors In Exercise 5, students may think the number is irrational. Point out that the decimal has a definite end at 9, so it can be written as the ratio $\frac{314{,}159}{100{,}000}$.

Teaching Strategy For Exercises 10, students should refer to the argument used in Example 2.

Answers

1. rational; It can be written as the ratio $\frac{3}{7}$.
2. rational; It is written as the ratio $\frac{5}{8}$.
3. irrational; It cannot be written as a ratio of integers.

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4. irrational; It cannot be written as a ratio of integers.
5. rational; It can be written as the ratio $\frac{314{,}159}{100{,}000}$.
6. rational; It can be written as a ratio $\frac{-8}{11}$.
7. rational; It can be written as the ratio $\frac{-11}{12}$.
8. irrational; It cannot be written as a ratio of integers.
9. irrational; It cannot be written as a ratio of integers.
10. Assume that $\sqrt{7}$ is a rational number, it can be written as the ratio of two integers p and q. $\frac{p}{q} = \sqrt{7}$. Assume that the integers p and q are both positive and chosen so that $\frac{p}{q}$ is in simplest form. $\frac{p^2}{q^2} = 7$, so $7q^2 = p^2$. So, p^2 is divisible by 7 and $p = 7m$ for a positive integer m. $7q^2 = (7m)^2 = 7^2m^2$. $q^2 = 7m^2$. So, q^2 is divisible by 7. $q = 7n$, for a positive integer n. Because 7 is factor p and 7 is a factor of q, $\frac{p}{q}$ is not in simplest form. This is a contradiction. So, $\sqrt{7}$ is irrational.
11. $\sqrt{4} = 2 = \frac{2}{1}$. So, $\sqrt{4}$ is rational.
12. Because π is an irrational number and $(2)^2$ is a rational number, the product is irrational.
13. Because both 6 and $\frac{3}{4}$ are rational numbers, the sum is rational.
14. Because both $\frac{3}{4}$ and $\frac{9}{10}$ are rational numbers, the difference is rational.
15. Because both $0.\overline{63} = \frac{7}{11}$ and $\frac{1}{3}$ are rational numbers, the sum is rational.
16. Because $\sqrt{37}$ is an irrational number and 1 is a rational number, the sum is irrational.
17. Because -6.5749 is a rational number and $\sqrt{15}$ is an irrational number, the sum is irrational.
18. Because $\left(\sqrt{3}\right)^2 = 3 = \frac{3}{1}$, $\left(\sqrt{3}\right)^2$ is rational.
19. Because $\frac{1}{5}$ is a rational number and $3.\overline{5} = 3\frac{5}{9}$ is a rational number, the product is rational.
20. $-\left(\sqrt{5^3}\right) = -\sqrt{125}$ is irrational because 125 is not a perfect square.
21. $\left(\frac{7}{16}\right)\left(\frac{2}{5}\right) = \frac{7}{40}$ is a rational number.
22. $\frac{3}{4} + \frac{4}{5} = \frac{31}{20}$ is a rational number.

23. Possible answer: $\sqrt{2}$; If $s = \sqrt{2}$, then $d = \sqrt{2} \cdot \sqrt{2} = \left(\sqrt{2}\right)^2 = 2 = \frac{2}{1}$, so *d* is a rational number.

24. Possible answer: 5; If $\ell = 5$ then $s = 2\sqrt{5 \cdot 5} = 2\sqrt{25} = 2(5) = 10 = \frac{10}{1}$, so *s* is a rational number.

Career Preparation: Check

Test-Taking Tip Students should pay attention for perfect square numbers under the square root sign. These numbers are rational numbers.

Answers

1. D	**2.** B	**3.** B	**4.** B
5. C	**6.** A	**7.** a, b, e, f	**8.** b, c, e
9. True, False, False, True, False		**10.** B	**11.** D

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Notes

LESSON 1.3

Applying Dimensional Analysis

CAREER SPOTLIGHT: Dental Laboratory Technician

Occupation Description

Dental laboratory technicians construct, fit, or repair crowns, bridges, dentures, and other dental appliances.

In small laboratories and offices, technicians may handle every phase of production. In larger ones, technicians may be responsible for only one phase of production, such as polishing, measuring, and testing.

Education

Dental laboratory technicians typically need a high school diploma or equivalent. There are some postsecondary programs in dental laboratory technology at community colleges or technical or vocational schools. High school students should take courses in science, human anatomy, math, computer programming, and art.

Potential Employers

The largest employers of dental laboratory technicians are as follows:

Medical equipment and supplies manufacturing	76%
Healthcare and social assistance	17%
Self-employed	4%
Government	2%

Watch a video about dental laboratory technicians: https://cdn.careeronestop.org/OccVids/OccupationVideos/51-9081.00.mp4

Career Cluster
Manufacturing

Career Pathway
Production

Career Outlook

- Salary Projections:
  Low-End Salary, $25,660
  Median Salary, $41,340
  High-End Salary, $65,820
- Jobs in 2018: 36,500
- Job Projections for 2028: 40,500 (increase of 11%)

Algebra Concepts

- Use units to understand problems.
- Use units to guide solutions to multi-step problems.

Is this a good career for me?

Dental laboratory technicians:

- Read work orders or other instructions to determine product specifications or materials requirements.
- Apply parting agents or other solutions to molds.
- Inspect medical or dental assistive devices.
- Mix ingredients to create specific finishes.

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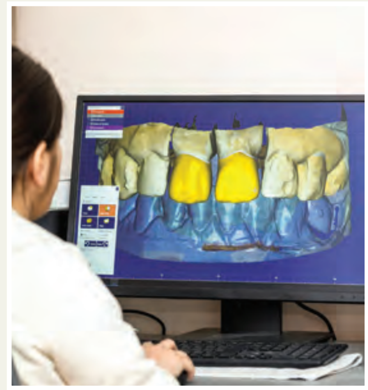
Lesson Objective

In this lesson, you will look at how the understanding of units and dimensions can help a dental laboratory technician solve problems.

1 Step Into the Career: Converting Units

A dental laboratory technician uses CAD (computer-aided design) to create a virtual model of a tooth. The actual width of the tooth is 9.15 millimeters. The image on the screen measures exactly six times the measurements made by the dentist. The technician will use the model to create a mold for a crown made from a polymer. Accounting for a 15% shrink in each dimension of the crown during the baking process, what should the width of the tooth mold be on the computer screen in inches? Use 1 in. = 2.54 cm.



Devise a Plan

Step 1: Convert the actual width in millimeters to the width on screen in millimeters.

Step 2: Convert the width on screen in millimeters to a width in inches.

Step 3: Find the width on screen of the tooth mold in inches.

Walk Through the Solution

Step 1: Convert the actual width in millimeters to the width on screen in millimeters. Since the width of the image on screen is six times the actual width, multiply by 6.

$$6 \cdot 9.15 \text{ mm} = 54.9 \text{ mm}$$

Step 2: Convert the measurement on screen in millimeters to a measurement in inches. Use the conversion factors 1 cm = 10 mm and 1 in. = 2.54 cm.

$$(54.9 \text{ mm})\left(\frac{1 \text{ cm}}{10 \text{ mm}}\right)\left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right) \approx 2.161 \text{ in.}$$

Step 3: Find the width of the tooth mold on screen in inches. Since there is 15% shrinkage, the width of the tooth on screen is 85% of the width of the tooth mold on screen. This means that the width of the tooth mold is the width of the tooth divided by 85%.

$$\frac{2.161 \text{ in.}}{85\%} = \frac{2.161 \text{ in.}}{0.85} \approx 2.5 \text{ in.}$$

The width of the tooth mold on the computer screen should be about 2.5 inches.

Photo credit: Bymandesigns/Shutterstock.com

On the Job: Apply Converting Units

1. A dental laboratory technician wants to try two new polymers to make a tooth crown. The actual width of the tooth is 8.26 millimeters.

   a. The screen image measures exactly five times the measurements made by the dentist. What will the width of the tooth model image be in millimeters on the computer screen? Write an equation, including units, to justify your answer.

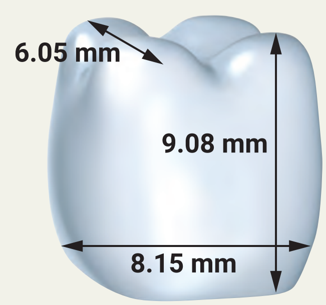
   b. What will the width of the tooth model image be in inches on the computer screen? Write an equation, including units, to justify your answer.

   c. The technician will use the model to create a mold for a crown. If the technician uses a new polymer that only shrinks 12% in each dimension during the baking process, what should the width of the tooth mold be in inches on the computer screen? Write equations, including units, to justify your answer.

   d. The second polymer that the technician wants to try only shrinks 4% in each dimension while baking. What should the width of the tooth mold be in centimeters on the computer screen? Write equations, including units, to justify your answer.

2 Step Into the Career: Percent Change in Volume

A dental laboratory technician is preparing to build a crown for a tooth. The actual width of the tooth is 8.15 millimeters. Its depth is 6.05 millimeters. The height of the tooth is 9.08 millimeters. The technician will use the model to create a mold for a crown to be made from a polymer. When the mold is baked from this polymer, there will be 15% of shrinkage in each dimension of the crown.



What dimensions should the technician make the mold before it is baked? What will the percent of shrinkage of the volume be if the length, width, and height each shrink by 15%?

Photo credit: Alex Mit/Shutterstock.com

Devise a Plan

Step 1: Determine how to write equations to calculate the measurements of the mold from the measurements of the patient's tooth.

Step 2: Calculate the dimensions of the mold before it is baked.

Step 3: Calculate the percent of shrinkage of the volume.

Step 4: Check the answer to Step 3 by calculating the volumes from the mold before and after it is baked. Then determine the percent of shrinkage, and compare the answer to your answer in Step 3.

Walk Through the Solution

Step 1: Since the shrinking happens from the mold before it is baked, the measurements of the patient's tooth is 85% of the measurements of the mold before it is baked. The measurements of the mold before it is baked is 100%. Write a proportion to relate the percentages to the measurements.

Step 2: Write the proportions, and then cross-multiply to find each unknown measurement.

$$\frac{85}{100} = \frac{8.15\text{ mm}}{x\text{ mm}};\ x = 8.15\text{ mm} \times \frac{100}{85} \approx 9.59\text{ mm}$$

$$\frac{85}{100} = \frac{6.05\text{ mm}}{x\text{ mm}};\ x = 6.05\text{ mm} \times \frac{100}{85} \approx 7.12\text{ mm}$$

$$\frac{85}{100} = \frac{9.08\text{ mm}}{x\text{ mm}};\ x = 9.08\text{ mm} \times \frac{100}{85} \approx 10.7\text{ mm}$$

Step 3: To calculate the percent of shrinkage of the volume, multiply the percent of shrinkage for each dimension.

$$\frac{85}{100} \cdot \frac{85}{100} \cdot \frac{85}{100} = \frac{61{,}4125}{1{,}000{,}000} \approx 0.61\text{, or } 61\%$$

Step 4: Approximate the shape of the tooth as a rectangular prism. Use the formula $V = (\text{length})(\text{width})(\text{height})$ to find the volume.

$$V_{\text{before}} = (9.59\text{ mm})(7.12\text{ mm})(10.7\text{ mm}) \approx 731\text{ mm}^3$$

$$V_{\text{after}} = (8.15\text{ mm})(6.05\text{ mm})(9.08\text{ mm}) \approx 448\text{ mm}^3$$

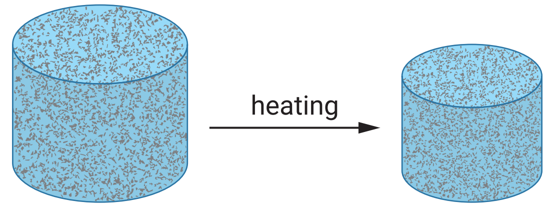
The percent of shrinkage can be found by dividing the after volume by the before volume.

$$\frac{V_{\text{after}}}{V_{\text{before}}} = \frac{448}{731} \approx 0.61\text{, or } 61\%$$

The results from Steps 3 and 4 agree.

On the Job: Apply Percent Change in Volume

2. A dental laboratory technician is studying various polymers to make crowns. He is studying their properties to find out which ones work best and which ones will cost more to use. Each dimension of Polymer A shrinks by 4% when heated by irradiation. Each dimension of Polymer B shrinks by 12%.



   a. What will the change in volume be for each one? Write equations to justify your answers.

   b. Determine the amount of each polymer needed to produce 100 milliliters after heating is applied. Write proportions, including units, to justify your answers.

   c. If the polymer that shrinks 4% costs $58 per milliliter and the polymer that shrinks 12% costs $38 per milliliter, which one costs less to produce 100 milliliters? Write equations, including units, to justify your answers.

3 Step Into the Career: Using Units in Proportions

A dental laboratory technician is preparing dental ceramic to make a crown. The technician uses prepared powders to make the ceramic. Each powder contains the ingredients kaolin and feldspar. The table shows the percentage of each ingredient in the ceramic produced by 100 grams of each powder. Dental ceramic should be composed of 3–5% kaolin and 70–85% feldspar. If the technician combines 70 grams of Powder A and 110 grams of Powder B, does the combined powder meet the requirements to produce dental ceramic? Assume that 100 grams of each powder produces the same amount of ceramic.

Powder	Kaolin	Feldspar
A	8%	70%
B	2%	90%

Devise a Strategy

Ask and answer the following questions to determine whether combining 70 grams of Powder A and 110 grams of Powder B can produce dental ceramic.

- **What do you know and do not know?**
  You know the percentages of kaolin and feldspar that 100 grams of each powder can produce, but you do not know how much dental ceramic is produced for 100 grams of each powder.
- **How can you find out whether the combined powder meets the requirements?**
  You need to find the percentages of kaolin and feldspar produced by the combined powder.
- **What can you conclude?**
  If the percentage of kaolin is between 3% and 5% and the percentage of feldspar is between 70% and 85%, then the combined powder meets the requirements.

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Walk Through the Solution

Step 1: Represent the amount of ceramic produced by 100 grams of each powder using a variable since it is unknown. Then use the variable with proportions to represent the amount produced by 70 grams of Powder A, the amount produced by 110 grams of Powder B, and the total amount produced by the combined powder.

Let x represent the number of grams of ceramic produced by 100 grams of each powder. Write out the units of measure to keep track.

The amount produced by 70 grams of Powder A is:

$$\frac{70 \text{ grams}}{100 \text{ grams}}(x \text{ grams}) = 0.7x \text{ grams}$$

The amount produced by 110 grams of Powder B is:

$$\frac{110 \text{ grams}}{100 \text{ grams}}(x \text{ grams}) = 1.1x \text{ grams}$$

The total amount produced by the combined powder is:

$0.7x$ grams + $1.1x$ grams = $1.8x$ grams

Step 2: Represent the amount of kaolin produced by each powder and by the combined powder.

Powder A: 8% of $0.7x$ grams = $(0.08)(0.7x$ grams$)$ = $0.056x$ grams

Powder B: 2% of $1.1x$ grams = $(0.02)(1.1x$ grams$)$ = $0.022x$ grams

Combined powder: $0.056x$ grams + $0.022x$ grams = $0.078x$ grams

Step 3: Represent the amount of feldspar produced by each powder.

Powder A: 70% of $0.7x$ grams = $(0.7)(0.7x$ grams$)$ = $0.49x$ grams

Powder B: 90% of $1.1x$ grams = $(0.9)(1.1x$ grams$)$ = $0.99x$ grams

Combined powder: $0.28x$ grams + $0.54x$ grams = $1.48x$ grams

Step 4: Determine the percentages of kaolin and feldspar produced by the combined powder.

Percentage of kaolin: $\dfrac{0.078x \text{ grams}}{1.8x \text{ grams}} \approx 0.043 = 4.3\%$

Percentage of feldspar: $\dfrac{1.48x \text{ grams}}{1.8x \text{ grams}} \approx 0.82 = 82\%$

The percentage of kaolin must be between 3% and 5%, and the percentage of feldspar must be between 70% and 85%. So, the combined powder meets the requirements to produce dental ceramic.

On the Job: Apply Using Units in Proportions

3. A dental laboratory technician is making a new toothpaste that must have 0.3% triclosan and 2.0% of a copolymer. The technician combines two powders. The table shows the percentages of triclosan and copolymer produced by 100 grams of each powder.

Powder	Triclosan	Copolymer
A	0.2%	1%
B	1.2%	5%

If the technician combines 50 grams of Powder A and 10 grams of Powder B, determine whether the combined powder produces a toothpaste that has 0.3% triclosan and 2.0% of the copolymer.

a. Using x to represent the grams of toothpaste produced by 100 grams of each powder, write an expression for the amount of toothpaste produced by the combined powders.

b. Using x, write expressions for the amount of triclosan produced by each powder and the combined powder. Include the units.

c. Write an equation for the amount of copolymer produced by each powder. Include the units.

d. Determine the percentages of triclosan and copolymer in the combined powder. Does the combined powder meet the requirements?

Career Spotlight: Practice

4. A dental laboratory technician is helping an archaeologist to study how much a tooth has been worn down in a 3 million-year-old *Australopithecus africanus* skull. The length of the tooth (including the root of the tooth) was 18 millimeters when it was found. The technician, using the structure of the tooth, was able to figure out that it had worn down about 3.4% from what it was originally. How long in millimeters would the tooth have been originally?

QUICK TIP

Sketching the situation can help you to figure out what mathematical steps are needed to solve. Draw a tooth to help you visualize.

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5. A dental laboratory technician is a making a mold with a polymer that shrinks by 9% in each dimension.

   a. How much does the surface area of the polymer shrink by?

   b. How much does the volume of the polymer shrink by?

6. A dental laboratory technician is making a ceramic tooth. The ceramic should have 68–72% lithium disilicate crystals and 3–5% kaolin. She combines two powders, A and B. The table shows the percentages of each ingredient produced by 100 grams of each powder. If she combines 40 grams of Powder A and 60 grams of Powder B, does the ceramic meet the required ranges of the percentages?

Powder	Kaolin	Lithium Disilicate
A	8%	80%
B	2%	65%

Devise a Plan

Step 1: Using *x* to represent the number of grams of ceramic produced by 100 grams of each powder, write the amount of ceramic that is produced by combining the powders as described.

Step 2: ______?______

Step 3: ______?______

Step 4: ______?______

7. While volunteering with Borderless Dentists in a foreign country, a dental laboratory technician encounters a supply of aspirin that is labeled in a way that is unfamiliar to her. She wants to give the patient a dose of about 0.5 to 0.7 grams to counteract the patient's pain, but the bottle is labeled in *grains* (gr). She looks up the unit in a pharmacy book and learns that one grain (1 gr) of aspirin is equal to 64.8 milligrams. If each aspirin tablet contains contains 5.0 grains of active aspirin, would two tablets provide the dosage the technician wants?

QUICK TIP

Begin with the simplest fact from the question, and then use unit conversion factors or ratios to find the dosage in milligrams. As you work through each step, be careful to make sure units properly cancel.

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Career Spotlight: Check

8. A dental laboratory technician is designing a replacement tooth using CAD. The actual width of the tooth to be replaced is 7.1 millimeters.

Select the answer from each box that makes the sentence true.

He is using a screen that expands the image eight times larger than the actual tooth. The width of the tooth on the screen will be

a. 0.9
b. 7.1
c. 57

millimeters.

The material used to make the model tooth shrinks by 18% when it is baked, so the model needs to be made larger than the actual tooth. The size of the model on the screen should be

a. 69
b. 47
c. 10

millimeters. Since 25.4 mm = 1 in., the size of the model on the screen is about

a. 27
b. 2.7
c. 1.9

inches.

9. A dental laboratory technician is making a full model of a patient's upper teeth so that the orthodontist can design braces to fit. She is using a mold that was made in the patient's mouth with a quick-hardening thermoplastic polymer. If the density of the material to make the model is 1.35 grams per cubic centimeter and the mass of the model is 17.2 grams, what is the volume of the model? Use 1 cm^3 = 1 mL.

A. 12.7 mL

B. 12.8 mL

C. 23.2 mL

D. 0.0784 mL

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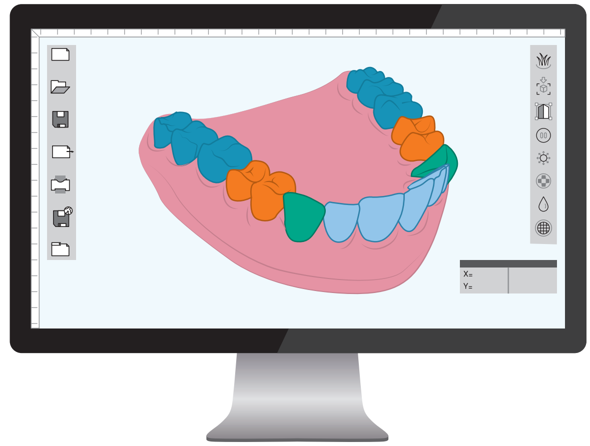
10. A dental laboratory technician is measuring out supplies at a clinic and is converting a dosage of pain medicine from a measurement in grains to a measurement in milligrams. The mass of one grain (1 gr) of aspirin is 0.0648 grams. One tablet of aspirin contains 2.5 grains. How many tablets make up the dose of baby aspirin, which is 81 milligrams? Write an equation including units to justify your answer.

11. An orthodontic technician is preparing a computer model of a patient's mouth using a mold that was made in the patient's mouth. The actual span across the back of the patient's lower molars is 61 millimeters. The technician uses CAD to create a virtual model of the patient's mouth on a computer screen. The model is 8 times larger than the actual mouth. The patient wants to know the measurements in inches.



Select all the statements that are true.
Use 1 in. = 2.54 cm and 10 mm = 1 cm.

a. The distance across the patient's lower molars on the screen would be about 7.5 millimeters.

b. The distance across the patient's lower molars on the screen would be about 49 centimeters.

c. The distance across the patient's actual lower molars is about 24 inches.

d. The distance across the patient's lower molars on the screen is about 19 inches.

e. The distance across the patient's lower molars on the screen is about 19 centimeters.

12. A dental laboratory technician is examining images of a patient's mouth on a computer to prepare to make a mold to model the mouth. The images are three times larger than the patient's actual mouth. The patient's upper teeth on the screen appear to be 17.43 centimeters across, from molars on one side to molars on the other. The mold will be made of a material that will shrink 10% during baking. Write an equation to determine the width of the mold in inches that could be used to make a model of the patient's teeth at actual size.

13. A dental laboratory technician has mislabeled the polymer containers in the lab. To find out what the polymer in one container is, the technician produces a cube of polymer and heats it. The polymer shrinks from an initial volume of 1.20 milliliters to a volume of 0.965 milliliters. Which polymer would it be?

A. Material A, which shrinks 8% in each dimension

B. Material B, which shrinks 4% in each dimension

C. Material C, which shrinks 7% in each dimension

D. Material D, which shrinks 14% in each dimension

14. A dental laboratory technician is studying new materials for tooth crowns in the lab. Material A shrinks 8% in each dimension when heated. Material B shrinks 4% in each dimension when heated. Material C shrinks 12% in each dimension when heated.

Match each material with the correct percent of shrinkage in surface area.

	Surface Area Shrink After Heating				
	8%	12%	15%	22%	23%
Material A, 8%	☐	☐	☐	☐	☐
Material B, 4%	☐	☐	☐	☐	☐
Material C, 12%	☐	☐	☐	☐	☐

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Notes

Teacher Edition

LESSON 1.3

Applying Dimensional Analysis

Common Core State Standards

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.

Mathematical Practices 4, 6, 7

CAREER SPOTLIGHT: Dental Laboratory Technician

Dental laboratory technicians use science and math to help produce various physical and chemical products needed by a dentist. They work with polymers and ceramics to build dentures, false teeth, and crowns. Some technicians develop proficiency in using computer-aided design (CAD) to make models of teeth. Dental laboratory technicians need to study anatomy, chemistry, physics, mathematics, and computing to be able to perform the tasks required in their job.

This career draws from principles in dentistry, engineering, computing, and chemistry.

- Discuss with students the work a dental laboratory technician will do by reading the Career Spotlight together.
- Find local colleges and universities with a dental technician program to share with students.
- Research local dental facilities that employ dental laboratory technicians, and ask what tasks the technicians perform.

Video: Dental Laboratory Technicians

Have students watch this video, which describes the types of projects a dental laboratory technician might work on.

Lesson Objective

In this lesson, you will look at how the understanding of units and dimensions can help a dental laboratory technician solve problems.

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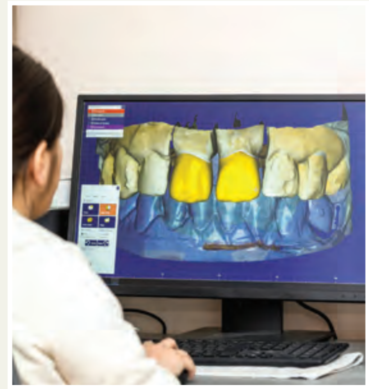
Teaching Support

1 Step Into the Career: Converting Units

A dental laboratory technician uses CAD (computer-aided design) to create a virtual model of a tooth. The actual width of the tooth is 9.15 millimeters. The image on the screen measures exactly six times the measurements made by the dentist. The technician will use the model to create a mold for a crown made from a polymer. Accounting for a 15% shrink in each dimension of the crown during the baking process, what should the width of the tooth mold be on the computer screen in inches? Use 1 in. = 2.54 cm.



Note: It may be helpful to write out Step 2 on a board, one factor at a time, so students see how the equation builds. Demonstrate how the units cancel. Cross out millimeters, then write the next factor, and cross out centimeters, so only inches are left.

Guiding Questions

- In Step 1, what math operation can you use to find the size on the screen?
- In Step 2, how do you know what conversion factors to use?
- In Step 3, how do you know whether the mold will be larger or smaller than the tooth?

DIFFERENTIATION: ADDITIONAL SUPPORT To demonstrate this concept, it may be helpful to model this scenario in a physical way so students can visualize the comparisons. For example, find two objects that look similar but have different sizes. One simple example would be to use two balls, such as a tennis ball and a volleyball. Use the tennis ball to represent a real tooth and the volleyball to represent the image on the screen. Having physical models may help kinesthetic learners.

Photo credit: Bymandesigns/Shutterstock.com

On the Job: Apply Converting Units

Answers

1a. $5 \cdot 8.26 \text{ mm} = 41.3 \text{ mm}$

1b. $(41.3 \text{ mm})\left(\dfrac{1 \text{ cm}}{10 \text{ mm}}\right)\left(\dfrac{1 \text{ in.}}{2.54 \text{ cm}}\right) \approx 1.626 \text{ in.}$

1c. $100\% - 12\% = 88\% = 0.88$

$$\frac{1.626 \text{ in.}}{0.88} \approx 1.8 \text{ in.}$$

1d. $\dfrac{1.626 \text{ in.}}{0.96} \approx 1.7 \text{ in.}$

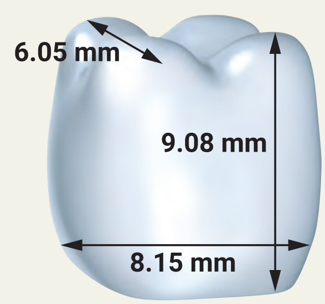
$$(1.7 \text{ in.})\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right) \approx 4.3 \text{ cm}$$

Use these questions to check students' understanding.

- In 1a, what is the unit of the tooth measurement? What is the unit of the screen measurement?
- In 1b, what did you calculate first?
- In 1c, why did you subtract 12% from 100%? What effect does dividing by a number less than one have?
- Did you need to do anything differently in solving 1d compared to 1c?

2 Step Into the Career: Percent Change in Volume

A dental laboratory technician is preparing to build a crown for a tooth. The actual width of the tooth is 8.15 millimeters. Its depth is 6.05 millimeters. The height of the tooth is 9.08 millimeters. The technician will use the model to create a mold for a crown to be made from a polymer. When the mold is baked from this polymer, there will be 15% of shrinkage in each dimension of the crown.



What dimensions should the technician make the mold before it is baked? What will the percent of shrinkage of the volume be if the length, width, and height each shrink by 15%?

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Guiding Questions

- In Steps 1 and 2, explain how you can determine which measurements correspond to 100% and which measurements correspond to 85% in the proportion.
- In Step 3, why is it possible to use just the percentages, rather than the actual measurements, to calculate the percent of shrinkage in volume?
- How could you find the percent of shrinkage in surface area using the same reasoning used in Step 3?
- What do the calculations in Step 4 show?

DIFFERENTIATION: ENRICHMENT Have students research how three-dimensional CAD can be used to model various body parts and design prosthetics. Encourage them to find out about the benefits and challenges of using different types of software and physical modeling methods. Students can investigate what kind of education is needed to become a computer-aided designer and the importance of learning about how the human body works in order to use CAD. Ask students whether someone could do a dental technician's work without fully understanding the anatomy of the mouth and the working of the jawbones and skull.

DIFFERENTIATION: ADDITIONAL SUPPORT Have students use two balls of different sizes and measure the radiuses to find the percent change from the larger ball to the smaller ball. Ask students to use the volume formula for spheres $\left(V = \frac{4}{3}\pi r^3\right)$ to find and compare the percent change between volumes. Then have them follow the method in Step 3 to find the percent change.

On the Job: Apply Percent Change in Volume

Answers

2a. Polymer A: $\frac{96}{100} \cdot \frac{96}{100} \cdot \frac{96}{100} = \frac{884{,}736}{1{,}000{,}000} \approx 88.5\%$ of the original volume

Polymer B: $\frac{88}{100} \cdot \frac{88}{100} \cdot \frac{88}{100} = \frac{681472}{1000000} \approx 68.1\%$ of the original volume

2b. Polymer A: $\frac{88.5}{100} = \frac{100 \text{ mL}}{x \text{ mL}}$; $x \approx 113$ mL

Polymer B: $\frac{68.1}{100} = \frac{100 \text{ mL}}{x \text{ mL}}$; $x \approx 147$ mL

2c. Polymer A: $113 \text{ mL} \cdot \frac{\$58}{1 \text{ mL}} = \$6554 \approx \$6600$

Polymer B: $147 \text{ mL} \cdot \frac{\$38}{1 \text{ mL}} = \$5586 \approx \$5600$

Polymer B costs less.

Use these questions to check students' understanding.

- In 2a, before you made any calculations, which polymer did you think would have a greater shrinkage in volume? How did you know?
- In 2b, how did you set up your proportions? Which polymer did you need to use more of?
- In 2c, what did you multiply to approximate the costs?

3 Step Into the Career: Using Units in Proportions

A dental laboratory technician is preparing dental ceramic to make a crown. The technician uses prepared powders to make the ceramic. Each powder contains the ingredients kaolin and feldspar. The table shows the percentage of each ingredient in the ceramic produced by 100 grams of each powder. Dental ceramic should be composed of 3–5% kaolin and 70–85% feldspar. If the technician combines 70 grams of Powder A and 110 grams of Powder B, does the combined powder meet the requirements to produce dental ceramic? Assume that 100 grams of each powder produces the same amount of ceramic.

Powder	Kaolin	Feldspar
A	8%	70%
B	2%	90%

Guiding Questions

- In Step 1, how do you know the unit of measure of the unknown x?
- In Step 1, explain in your own words what each part of the equation $0.7x$ grams + $1.1x$ grams = $1.8x$ grams represents.
- In your own words, what do the algebraic expressions in Steps 2 and 3 represent?
- In Step 4, why is it not necessary to solve for x in order to answer the question?

LANGUAGE SUPPORT Dental technology uses many ingredients with names that are unfamiliar to students. It may be helpful for students to substitute abbreviations for the names so they can more easily read the sentences. For example, students can replace kaolin and feldspar with K and F in the problem.

TECHNOLOGY If a technician were faced with this problem at work and did not know how to solve it using algebra, he or she could solve the problem by writing different combinations of Powder A and Powder B using spreadsheets. Demonstrate for students how a spreadsheet can easily compute percentages for different combinations of powders.

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On the Job: Apply Using Units in Proportions

Answers

3a. $0.6x$ grams

3b. Powder A: $0.001x$ g, Powder B: $0.0012x$ g, combined powder: $0.0022x$ g

3c. Powder A: $0.005x$ g, Powder B: $0.005x$ g, combined powder: $0.01x$ g

3d. triclosan: 0.4%, copolymer: 1.7%

These amounts are close to, but do not meet, the desired value for the toothpaste. The dental technician will need to make adjustments to meet the requirements.

Use these questions to check students' understanding.

- In 3a, how did you find expressions for each powder?
- In 3b, how did you find expressions for each powder?
- In 3c, which expression did you use to find the percentages?

Career Spotlight: Practice

Solution Steps for Exercises 4–7

These steps will help guide students in solving these practice exercises.

Exercise 4

Answer

4. 18.63 mm ≈ 19 mm

Solution Steps

- Let x be the original length. Write a proportion. $\left(\frac{x \text{ mm}}{18 \text{ mm}} = \frac{100}{96.6}\right)$
- Solve the proportion. (about 18.63 mm)

Exercise 5

Answers

5a. 17%

5b. 25%

Solution Steps

- Find the percentage of the surface area of the mold after baking compared to surface area of the mold before baking. (about 83%)
- Find the percentage of the volume of the mold after baking compared to the volume of the mold before baking. (about 75%)

Exercise 6

Answer

6. The combined powder produces a ceramic with 4.4% kaolin and 71% lithium disilicate, so the ceramic meets the required ranges of the percentages.

Devise a Plan

Possible plan:

Step 1: Using x to represent the number of grams of ceramic produced by 100 grams of each powder, write the amount of ceramic that is produced by combining the powders as described.

Step 2: Write an expression to represent the amount of kaolin produced by each powder and the combined powder. Include the units.

Step 3: Write an expression to represent the amount of lithium disilicate produced by each powder and the combined powder. Include the units.

Step 4: Determine the percentages of kaolin and lithium disilicate in the combined powder.

Solution Steps

- Using x to represent the number of grams of ceramic produced by 100 grams of each powder, write the amount of ceramic that is produced by combining the powders as described. (x grams)
- Write the amount of kaolin produced by each powder and the combined powder. Include the units. (Powder A: $0.032x$ g, Powder B: $0.012x$ g, combined powder: $0.044x$ g)
- Write the amount of lithium disilicate produced by each powder and the combined powder. Include the units. (Powder A: $0.32x$ g, Powder B: $0.39x$ g, combined powder: $0.71x$ g)
- Determine the percentages of kaolin and lithium disilicate in the combined powder. (kaolin: 4.4%, lithium disilicate: 71%)

Exercise 7

Answer

7. There are about 0.65 grams of active ingredient in two aspirin tablets, so yes, the dose matches.

Solution Steps

- Begin with the simplest unit, 5.0 grains. Determine the conversion factor to use. $\left(\frac{64.8 \text{ mg}}{1 \text{ gr}}\right)$
- Next, determine the conversion factor to use to convert milligrams to grams. $\left(\frac{1 \text{ g}}{1000 \text{ mg}}\right)$
- Write the expression using the two conversion factors, and find the number of grams to compare. $\left(5.0 \text{ gr} \cdot \frac{64.8 \text{ mg}}{1 \text{ gr}} \cdot \frac{1 \text{ g}}{1000 \text{ mg}} \cdot 2 \text{ tablets} \approx 0.65 \text{ g}\right)$

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Career Spotlight: Check

Tips for Completing Exercises 8–14

These tips will help students in solving these exercises and similar assessment items.

Exercise 8

Answer

8. c. 57, a. 69, b. 2.7

Tip Encourage students to make a sketch to help visualize the situation. Have students check their answers for reasonableness—the number of millimeters is about 25 times the number of inches, so the number of inches should be less than the number of millimeters.

Exercise 9

Answer

9. A

Tip Remind students to pay attention to the units when writing an equation to solve this problem. Because the answer is in milliliters, students must divide by the density, not multiply. Students should check that the units properly cancel.

Exercise 10

Answer

10. $(81\text{ mg})\left(\frac{1\text{ gr}}{64.8\text{ mg}}\right)\left(\frac{1\text{ tablet}}{2.5\text{ gr}}\right) = 0.50\text{ tablet}$

Tip Encourage students to make sure that their answer is reasonable. Have them write all the conversions as fractions and make sure that the units properly cancel.

Exercise 11

Answer

11. b and d

Tip It can be easier to answer this type of question by first solving for each measurement in millimeters and then converting to match units in the options rather than looking at the options first and then trying to make sense of which ones will match the problem.

Exercise 12

Answer

12. $(17.43\text{ cm})\left(\frac{1}{3}\right)\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right)\left(\frac{100}{90}\right) \approx 2.54\text{ in.}$

Tip Encourage students to sketch the situation to help make sure that their answer makes sense. If writing a single equation is confusing, it can be helpful to work out the calculation in steps, making sense of each step along the way, and then combine the steps into a single equation.

Exercise 13

Answer

13. C

Tip Students should first determine the percent of shrink of the volume. First, divide the final volume by the initial volume to find out that it has been reduced to 80.4% of the original. Next, students can either take the cube root of 0.804 or work by trial and error to find out which percent of shrink would give a shrink in volume of 19.6%. The answer can be found by multiplying $1 - 0.07 = 0.93$ three times, $(0.93)(0.93)(0.93) = 0.804$.

Exercise 14

Answer

14. Material A, 8%: 15%, Material B, 4%: 8%, Material C, 12%: 23%

Tip Remind students that to calculate area, they multiply two dimensions. This means they will square the percent left after the reduction to determine the percent of area remaining. Students should also check that the order of shrinkage in surface area from least to greatest should match the shrinkage in each dimension from least to greatest.

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Notes