PATHWAY2CAREERS
GEOMETRY

# Pathway2Careers
# Geometry

PATHWAY2CAREERS
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## About Pathway2Careers

As NS4ed's flagship program, Pathway2Careers (P2C) is a career readiness solution that support schools in their efforts to prepare students for high-value careers in their communities. P2C delivers access to labor market data to encourage the use of data-informed practices in career education through a number of services, including labor market data tools, career-focused curricula, Perkins V support, and more. As a comprehensive career readiness solution, P2C serves administrators, faculty, students, and parents in making informed decisions that promote employment success.

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# Pathway2Careers Geometry

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| | | | |
|---|---|---|---|
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| | | | |
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| | | | |
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LESSON 1.1

# Points, Lines, and Planes

## CAREER PREPARATION: Essential Geometry Skills

### Did you know?

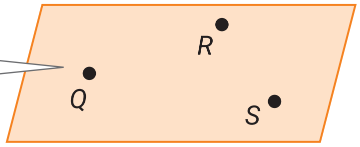
Woodworkers use properties of points, lines, and planes to design and build furniture.

### Consider this problem…

A woodworker is testing different designs for a table. He discovers that a table with four legs sometimes wobbles, but a table with three legs is always stable.

Which postulate justifies his discovery?

The floor represents a plane, and the bottoms of the legs of the table represent points.




**Let's find out…** about properties of points, lines, and planes so you can explain the woodworker's discovery on your own in the **Career Preparation Exercises.**

### Lesson Objective

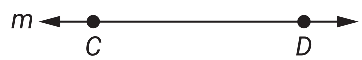
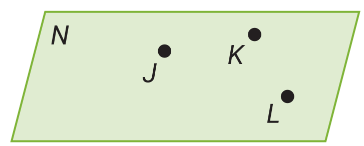
In this lesson, you will understand points, lines, and planes.

- You will identify, name, and draw points, lines, segments, rays, and planes.
- You will sketch intersections of lines and planes.

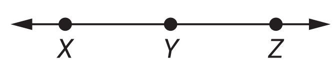
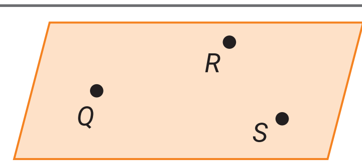
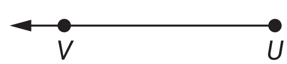
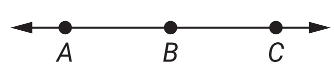
Photo credit: PR Image Factory/Shutterstock.com; ALPA PROD/Shutterstock.com

## Geometry Essentials

In geometry, **undefined terms** are basic figures that are described but not formally defined. Three undefined terms are point, line, and plane.

| Description | Diagram | Words and Symbols |
|---|---|---|
| A **point** represents a location. It has no size or shape. |   | point *A* |
| A **line** is an infinite number of points on a straight path that extends forever in opposite directions. It has no thickness. |   | line *m*, $\overleftrightarrow{CD}$, or $\overleftrightarrow{DC}$ |
| A **plane** is an infinite number of points on a flat surface that extends forever. It has no thickness. |   | plane *N*, plane *JKL* |

Other terms can be defined using points, lines, and planes.

| Definition | Diagram | Words and Symbols |
|---|---|---|
| **Collinear** points are points that lie on the same line. |   | Points *X*, *Y*, and *Z* are collinear. |
| **Coplanar** points are points that lie on the same plane. |   | Points *Q*, *R*, and *S* are coplanar. |
| A **segment** is part of a line consisting of two **endpoints** and all points between them. |   | Points *G* and *H* are the endpoints of $\overline{GH}$. |
| A **ray** is part of a line consisting of an **endpoint** and all points on one side of the endpoint. |   | Point *U* is the endpoint of $\overrightarrow{UV}$. |
| **Opposite rays** are two rays that have the same endpoint and together form a line. |   | $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. |

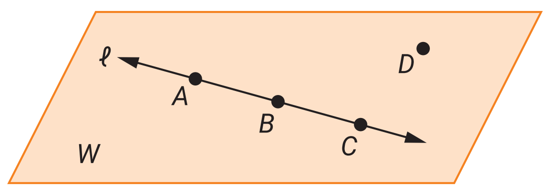
A **postulate** is a statement that is accepted without proof. There are five postulates related to points, lines, and planes.

| **Point, Line, and Plane Postulates** | |
|---|---|
| Postulate 1.1 | Through any two points, there is exactly one line. |
| Postulate 1.2 | If two lines intersect, then they intersect at exactly one point. |
| Postulate 1.3 | Through any three points that are not collinear, there is exactly one plane. |
| Postulate 1.4 | A plane contains at least three non-collinear points. |
| Postulate 1.5 | If two planes intersect, then their intersection is a line. |

## Identifying, Naming, and Drawing Points, Lines, Segments, Rays, and Planes

### Example 1 Naming Points, Lines, Segments, Rays, and Planes

**Use the figure to name each of the following.**




**a.** three points

**b.** one line named two different ways

**c.** two segments

**d.** two rays

**e.** one plane named two different ways

**Solution**

**a.** The figure shows four points: point *A*, point *B*, point *C*, and point *D*.

**b.** The line shown in the figure can be named line $\ell$ or using any two points on the line: $\overleftrightarrow{AB}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CB}$, $\overleftrightarrow{AC}$, or $\overleftrightarrow{CA}$.

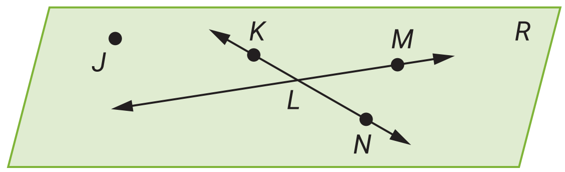
**c.** Any two points on the line can be the endpoints of a segment: $\overline{AB}$, $\overline{BA}$, $\overline{BC}$, $\overline{CB}$, $\overline{CA}$, or $\overline{AC}$.

**d.** To name a ray, name the endpoint of the ray and a point that the ray passes through: $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{CB}$, or $\overrightarrow{CA}$.

**e.** The plane shown can be named plane *W* or using any three noncollinear points on the plane: plane *ABD*, plane *ACD*, or plane *BCD*.

## Example 2 Identifying Collinear and Coplanar Points and Opposite Rays

**Use the figure to identify each of the following.**




**a.** three collinear points **b.** four coplanar points **c.** a pair of opposite rays

**Solution**

**a.** Since $\overleftrightarrow{KN}$ contains point *L*, points *K*, *L*, and *N* are collinear.

**b.** Plane *R* contains points *J*, *K*, *L*, *M*, and *N*. So, any four of points *J*, *K*, *L*, *M*, and *N* are four coplanar points.

**c.** Point *L* is the endpoint of $\overrightarrow{LK}$ and $\overrightarrow{LN}$, and $\overrightarrow{LK}$ and $\overrightarrow{LN}$ together form a line. Therefore, $\overrightarrow{LK}$ and $\overrightarrow{LN}$ are opposite rays.

## Example 3 Drawing Points, Lines, Segments, Rays, and Planes

**Draw each of the following.**

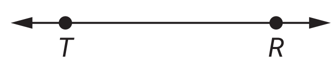
**a.** point *X* **b.** $\overleftrightarrow{TR}$ **c.** $\overline{MN}$

**d.** $\overrightarrow{PQ}$ **e.** plane *DEF*

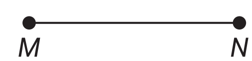
**Solution**

**a.** Draw a point and label it *X*.


**b.** Draw a line that contains points *T* and *R*.

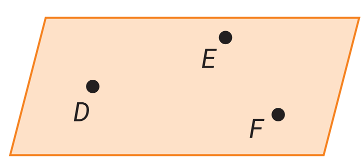



**c.** Draw a segment with endpoints *M* and *N*.




**d.** Draw a ray with endpoint *P* and containing point *Q*.

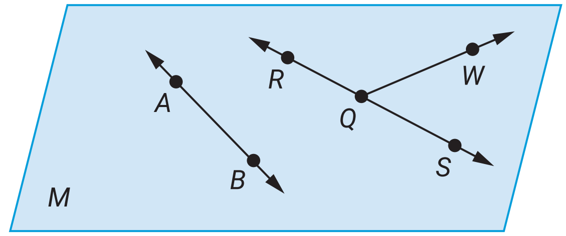

**e.** Draw a plane that contains points *D*, *E*, and *F*.



## Example 4 Drawing Figures Using Points, Lines, Segments, Rays, and Planes

**Draw plane $M$ that contains $\overleftrightarrow{AB}$, opposite rays $\overrightarrow{QR}$ and $\overrightarrow{QS}$, point $W$, and $\overrightarrow{QW}$.**

**Solution**




**Step 1:** Begin by drawing plane *M*.

**Step 2:** Draw a line on the plane and draw points *A* and *B* on the line.

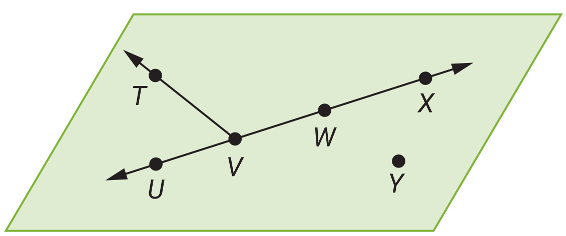
**Step 3:** Since rays $\overrightarrow{QR}$ and $\overrightarrow{QS}$ are opposite rays, they form a line. Draw another line on the plane and place points *R*, *Q*, and *S* on the line forming rays $\overrightarrow{QR}$ and $\overrightarrow{QS}$.

**Step 4:** Draw point *W* on the plane.

**Step 5:** Finally, draw $\overrightarrow{QW}$ by connecting points *Q* and *W* with an arrow after *W*.

## Build Your Skills: Try Identifying, Naming, and Drawing Points, Lines, Segments, Rays, and Planes

**In Exercises 1–5, use the diagram to name or identify an example of each type of figure.**




1. a segment
2. a plane
3. a ray
4. opposite rays
5. three collinear points
6. Draw plane *ABC* that contains $\overline{FG}$ and $\overrightarrow{MN}$.

### Did you know?

*Graphic designers use points, segments, and planes to design logos.*

## Geometry Essentials

The intersection of two or more geometric figures is the set of points that the figures have in common. Examples of intersections are listed below.

- Postulate 1.2 states that if two lines intersect, then they intersect at a point.
- If a plane and a line that is not on the plane intersect, their intersection is a point.
- Postulate 1.5 states that if two planes intersect, their intersection is a line.
- If two planes do not intersect, they are called **parallel planes**.

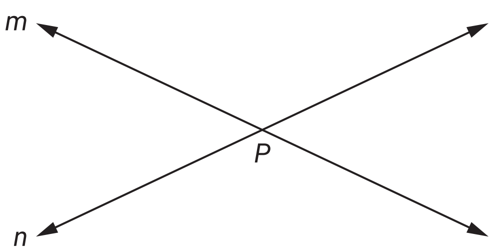
## Sketching Intersections of Lines and Planes

### Example 5 Sketching the Intersection of Two Lines

**Sketch lines *m* and *n* that intersect at point *P*.**

**Solution**

Draw two lines. Label them *m* and *n*. Label the point where the lines intersect as point *P*.





### Example 6 Sketching the Intersection of a Line and a Plane

**Sketch $\overleftrightarrow{GH}$ and plane *N* that intersect at point *C*.**

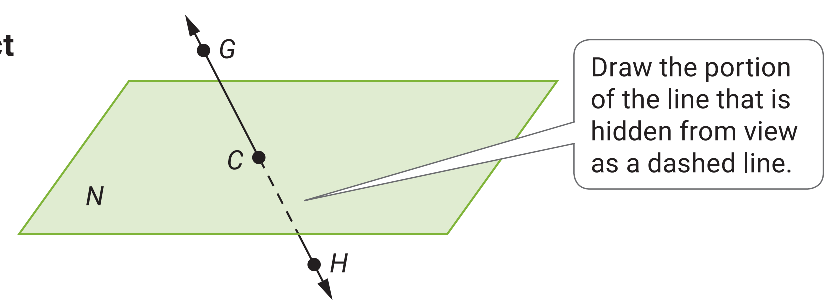
**Solution**

**Step 1:** Draw plane *N*.

**Step 2:** Draw a line that passes through the plane.

**Step 3:** Mark points *G* and *H* on the line.

**Step 4:** Label the point where the line passes through the plane as point *C*.



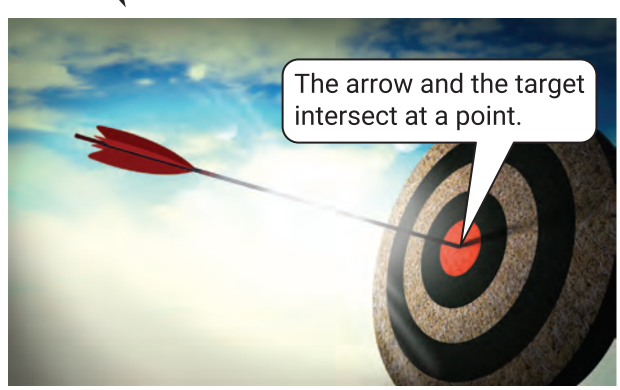




Photo credit: B Brown/Shutterstock.com; STUDIO DREAM/Shutterstock.com

## Example 7 Sketching the Intersection of Two Planes

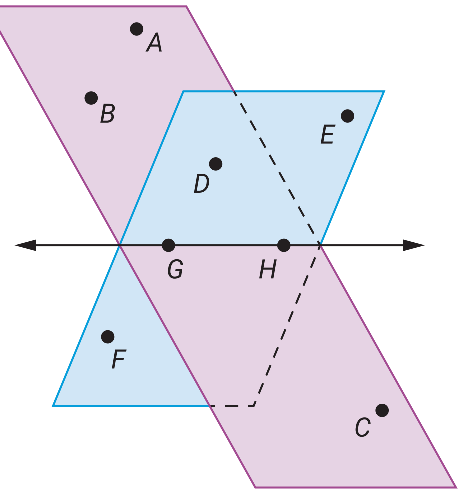
**Sketch planes *ABC* and *DEF* that intersect at $\overleftrightarrow{GH}$.**

**Solution**

**Step 1:** Draw a plane with points *A*, *B*, and *C*.

**Step 2:** Draw a second plane with points *D*, *E*, and *F*. Use dashed lines for portions of the plane edges that are hidden from view.

**Step 3:** Draw points *G* and *H* on the line where the two planes intersect.





## Build Your Skills: Try Sketching Intersections of Lines and Planes

7. Sketch $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ that intersect at point *M*.

8. Sketch $\overleftrightarrow{QR}$ and $\overleftrightarrow{ST}$ that do not intersect.

9. Sketch plane *R* and line *b* that intersect at point *Z*.

10. Sketch plane *JKL*, line *a* and line *b*, where line *a* lies on plane *JKL*, line *b* does not lie on plane *JKL*, and lines *a* and *b* intersect at point *T*.

11. Sketch plane *A* and plane *B* that intersect at line *m*.

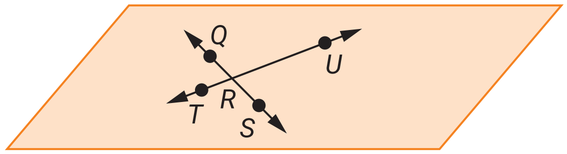
### Did you know?

*Fine artists use lines that intersect at one point, called a vanishing point, to show depth in a painting or drawing.*

Photo credit: Photographee.eu/Shutterstock.com; Valery Rybakow/Shutterstock.com

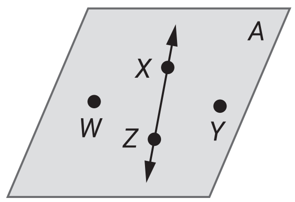
## Career Preparation: Practice

**Use the diagram to give an example of each figure.**

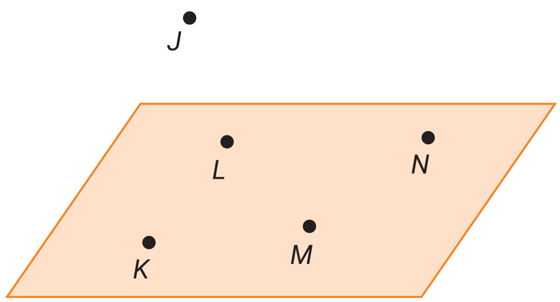



1. one point
2. one segment
3. one ray
4. one line

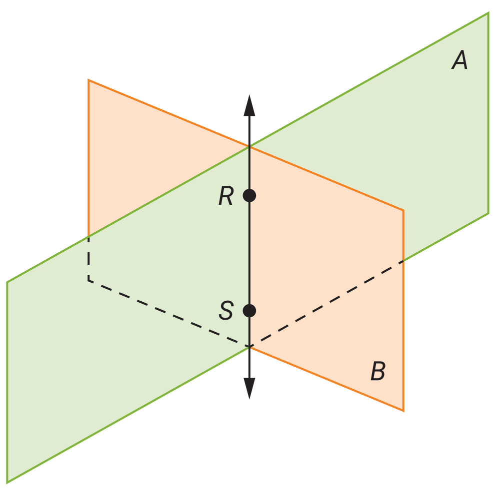
5. Name the plane shown in three different ways.




6. Identify four points that are coplanar and four points that are not coplanar.




7. Name the intersection of plane *A* and plane *B*.



8. Draw $\overrightarrow{WX}$ and $\overrightarrow{WY}$, where $\overrightarrow{WX}$ and $\overrightarrow{WY}$ are not opposite rays.

9. Draw four coplanar points where exactly three of the points are collinear.

10. Draw plane *A* that contains $\overline{PQ}$ and $\overrightarrow{GH}$.

**Sketch the figure described.**

11. planes *ABC* and *DEF* that intersect at $\overrightarrow{GH}$

12. lines *a* and *b* and plane *R* so that line *a* and plane *R* intersect at point *W*, line *a* and line *b* intersect at point *Q*, and line *b* and plane *R* do not intersect

13. lines *p*, *q*, and *r* that all intersect at point *T*

14. **Always-Sometimes-Never** Choose the appropriate word to complete the following statement: A line (always, sometimes, never) contains an infinite number of pairs of opposite rays. Explain your answer.

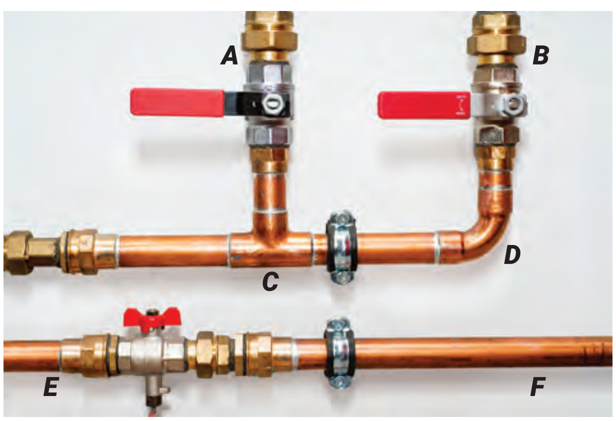
15. **Error Analysis** Your friend says that two planes will always intersect to form a line. Explain why your friend is incorrect and draw an example to prove your reasoning.

16. Is it possible that the same three points could be contained in two different planes? Explain.

17. **Writing** Describe a real-life example of a plane, a line, and their point of intersection.

## Use It On the Job

18. Tina is a plumber who has just installed the pipes shown in a new apartment building.




Name a line, a segment, and a ray from the diagram.

19. Neil is a woodworker. He designs and builds several different tables, and he discovers that a table with four legs sometimes wobbles, but a table with three legs is always stable.

Which postulate justifies his discovery?

Photo credit: Algirdas Gelazius/Shutterstock.com; ALPA PROD/Shutterstock.com

## Career Preparation: Check

1. Which geometric figure best represents the light from a flashlight?

   **A.** a point
   **B.** a line
   **C.** a plane
   **D.** a ray

2. Which geometric figure best represents a soccer field?

   **A.** a segment
   **B.** a line
   **C.** a plane
   **D.** a point

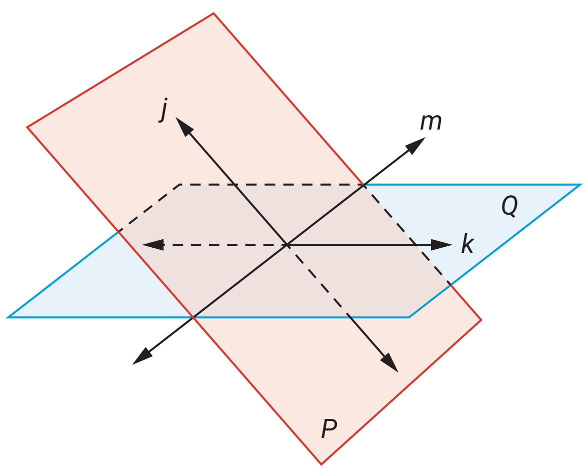
3. Which statement best explains why any two points are collinear?

   **A.** A point has no size or shape.
   **B.** Through any two points, there exists exactly one line.
   **C.** A line contains at least two points.
   **D.** A line contains an infinite number of points.

4. Which statement best explains why a plane can be named using three noncollinear points on the plane?

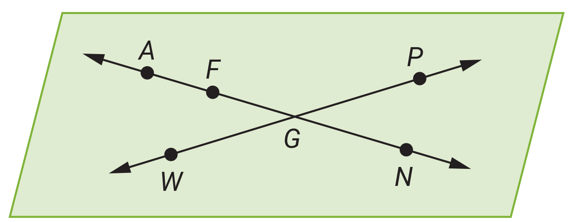
   **A.** A plane contains an infinite number of points.
   **B.** A plane contains at least three points.
   **C.** Through any three points that are not collinear, there is exactly one plane.
   **D.** A plane extends forever.

5. Select all the statements that are true about the figure.



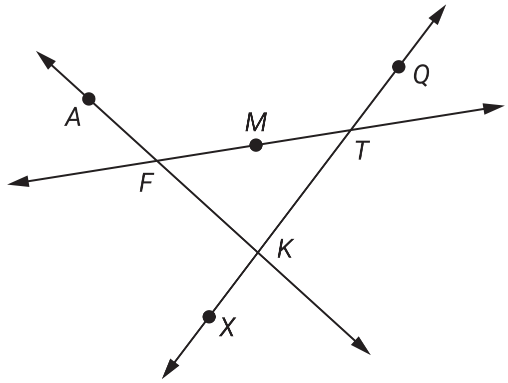

   **a.** Plane $P$ contains line $j$.
   **b.** Plane $P$ contains line $k$.
   **c.** Plane $P$ contains line $m$.
   **d.** Plane $Q$ contains line $j$.
   **e.** Plane $Q$ contains line $k$.
   **f.** Plane $Q$ contains line $m$.

6. Select all the statements that are true about the figure.




   **a.** Points $A$, $F$, and $W$ are coplanar.
   **b.** $\overrightarrow{GP}$ and $\overrightarrow{GN}$ are opposite rays.
   **c.** $\overrightarrow{FA}$ and $\overrightarrow{GN}$ are opposite rays.
   **d.** $\overleftrightarrow{AN}$ and $\overleftrightarrow{PW}$ intersect at point $G$.
   **e.** All points on $\overline{FG}$ are also on $\overleftrightarrow{AN}$.
   **f.** $\overleftrightarrow{PW}$ and plane $AFW$ do not intersect.

7. Which set of points is collinear?



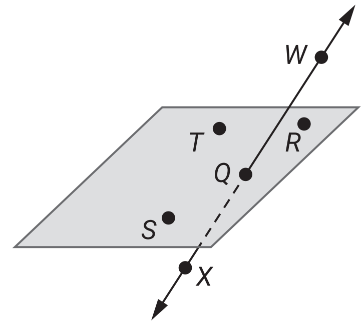

**A.** *F*, *T*, *K*

**B.** *K*, *M*, *Q*

**C.** *A*, *Q*, *X*

**D.** *F*, *M*, *T*

8. What is the intersection of plane *RST* and $\overleftrightarrow{WX}$?




**A.** point *Q*

**B.** point *X*

**C.** $\overleftrightarrow{WX}$

**D.** plane *RST*

## Use It On the Job

9. Kyle is an architect. He makes a model of a house he is designing.

Kyle uses planes to represent the walls of the house. What type of figure represents the intersections of the walls?

**A.** a point

**B.** a line

**C.** a plane

**D.** a ray

10. A city planner is planning a new subdivision. Three main roads will pass through the subdivision, and the roads are straight. The planner wants to place a set of traffic lights at each point where main roads intersect. What is the maximum possible number of intersections where traffic lights will be needed?

**A.** 1

**B.** 2

**C.** 3

**D.** 4

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# Notes

LESSON 1.1

# Points, Lines, and Planes

# Teacher Edition

## CAREER PREPARATION: Essential Geometry Skills

### Common Core State Standards

**G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and a distance around a circular arc.

**Mathematical Practices** 3, 4, 5

### Career Connections

The skills taught in this lesson will be applied in the following career-focused lessons:

- Lesson 1.8, Midpoint and Distance in the Coordinate Plane, which focuses on emergency medical technicians
- Lesson 1.9, Perimeter in the Coordinate Plane, which focuses on fence erectors
- Lesson 1.10, Area in the Coordinate Plane, which focuses on computer specialists

### Lesson Objective

In this lesson, you will understand points, lines, and planes.

- You will identify, name, and draw points, lines, segments, rays, and planes.
- You will sketch intersections of lines and planes.

### Vocabulary

- undefined terms
- point
- line
- plane
- collinear
- coplanar
- segment
- endpoint
- ray
- opposite rays
- postulate
- parallel planes

# Teaching Support

## Geometry Essentials

**Use Tools** Some students may benefit from using manipulatives, such as pieces of cardboard to represent planes and pencils, dowels, or pieces of uncooked spaghetti to represent lines. Students can represent points on these manipulatives using a marker.

**Vocabulary** Explain that although dictionaries contain definitions for the terms that are *undefined terms,* there is no mathematical definition, because they cannot be defined using other mathematical terms. Instead, these terms are described, and examples are given. The difference between descriptions and definitions may seem subtle to students.

**Teaching Strategy** Make sure that students understand that this lesson introduces basic geometric concepts which form the foundation of the study of geometry.

## Identifying, Naming, and Drawing Points, Lines, Segments, Rays, and Planes

**Teaching Strategy** Connect the naming of lines and planes to Postulates 1.1 and 1.3. Since two points determine a line, a line may be named using any two points on the line. Since three noncollinear points determine a plane, a plane may be named using any three noncollinear points on the plane.

**Avoid Common Errors** When naming rays, make sure that students name the endpoint first, no matter the orientation of the ray.

### Example 2 Identifying Collinear and Coplanar Points and Opposite Rays

**Check Understanding** For part b, ask students whether any three of the points on the plane could be used to name the plane. Make sure students understand that the three points used to name the plane cannot be collinear.

### Example 4 Drawing Figures Using Points, Lines, Segments, Rays, and Planes

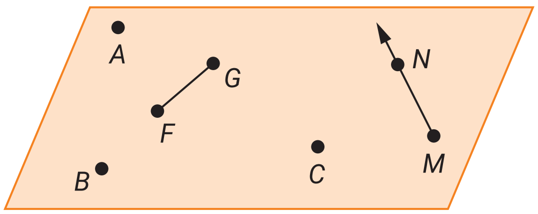
**Teaching Strategy** Point out to students that to draw a plane, they should draw a non-rectangular parallelogram. This is the standard figure used to represent a plane because it shows three-dimensional perspective.

**Check Understanding** Ask students whether $\overrightarrow{QR}$ and $\overrightarrow{QW}$ are opposite rays. Although the rays have a common endpoint, they do not form a line, so they are not opposite rays.

## Build Your Skills: Try Identifying, Naming, and Drawing Points, Lines, Segments, Rays, and Planes

**Answers**

1. Possible answer: $\overleftrightarrow{VW}$
2. Possible answer: plane *TXY*
3. Possible answer: $\overleftrightarrow{VT}$
4. Possible answer: $\overrightarrow{VX}$ and $\overrightarrow{VU}$
5. Possible answer: *U*, *V*, and *W*
6. Possible answer:




## Geometry Essentials

**Use Tools** Before reading the information in this section, give students time to explore intersections of two lines, a line and a plane, and two planes using manipulatives. Then, as you read each statement, have students display what the statement means using the manipulatives.

## Sketching Intersections of Lines and Planes

**Teaching Strategy** Point out the real-world examples that represent intersecting lines, an intersecting line and plane, and two intersecting planes. Ask students for other real-world examples of these intersections.

### Example 6 Sketching the Intersection of a Line and a Plane

**Check Understanding** Ask students if a line and a plane always intersect at a point. Make sure they understand that if the line is contained in the plane, the intersection is the line.

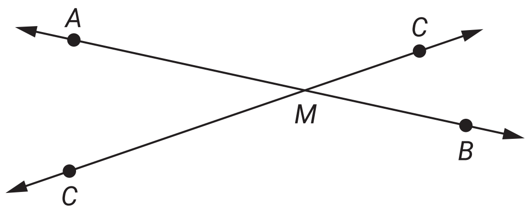
### Example 7 Sketching the Intersection of Two Planes

**Avoid Common Errors** Drawing a three-dimensional figure in two dimensions is challenging for students. Consider cutting slits in two pieces of colored construction paper to model the intersecting planes. Bring students' attention to which portions of which planes are visible so that they correctly shade the two planes in their diagram and use dashed lines for edges that are not visible.

## Build Your Skills: Try Sketching Intersections of Lines and Planes

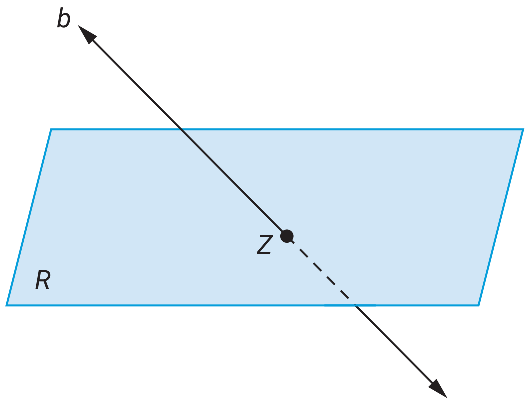
**Answers**

**7.** Possible answer:

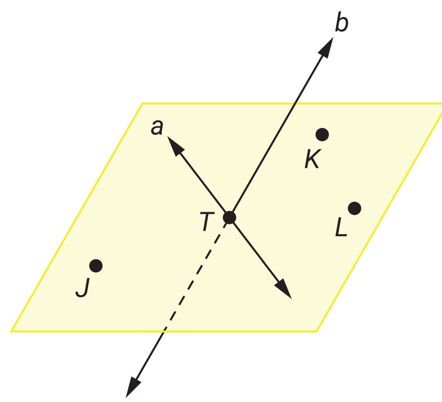



**8.** Possible answer:

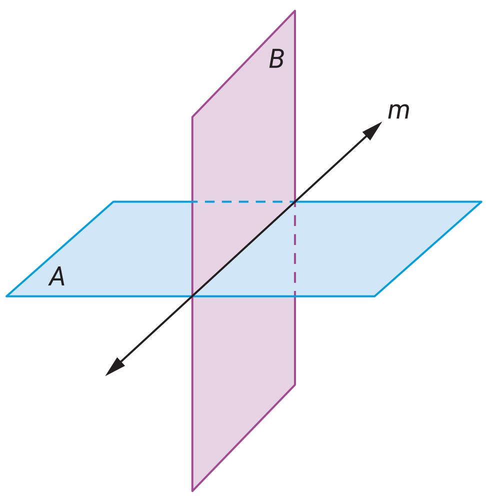

**9.** Possible answer:




**10.** Possible answer:




**11.** Possible answer:



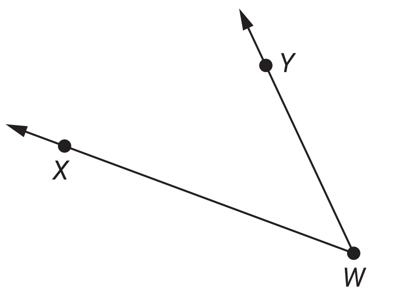

## Career Preparation: Practice

**Avoid Common Errors** Students should be careful to use the correct notation for rays, segments, and lines. The endpoint and point for a ray should have an arrow over them, with the endpoint named first. A segment should have an overbar above the segment endpoints. A line should have an arrow above the two points.

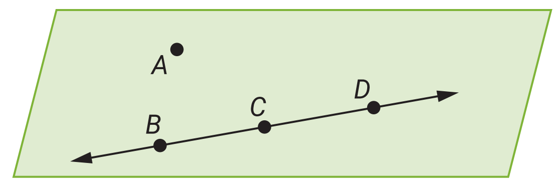
**Teaching Strategy** When sketching figures, encourage students to read the description twice before drawing the sketch. After drawing the sketch, have them reread the description to make sure the description and sketch match.

**Answers**

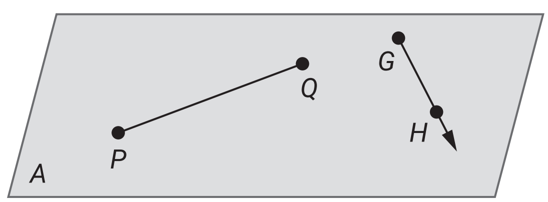
1. Possible answer: *T*
2. Possible answer: $\overline{RS}$
3. Possible answer: $\overrightarrow{RQ}$
4. Possible answer: $\overleftrightarrow{TU}$
5. Possible answer: plane *A*, plane *XYZ*, plane *WXZ*
6. coplanar points: *K*, *L*, *M*, *N;* not coplanar points: Possible answer: *J*, *K*, *L*, *M*
7. $\overleftrightarrow{RS}$
8. Possible answer:



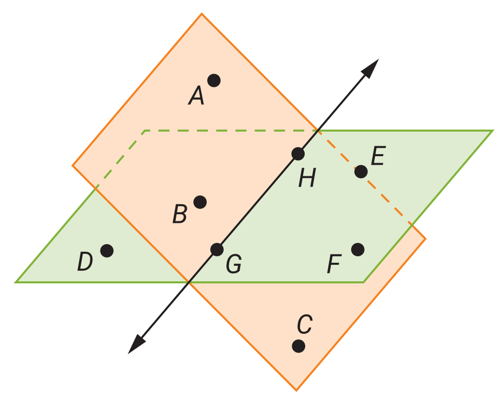

9. Possible answer:



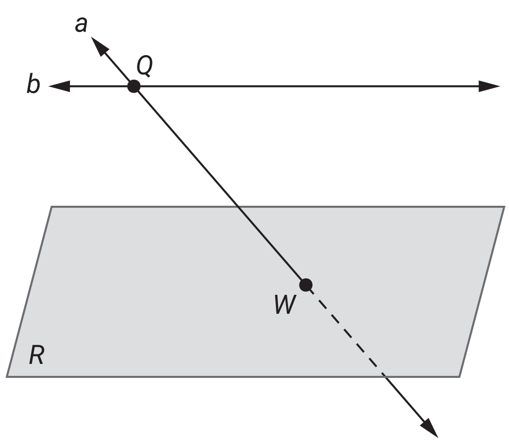

10. Possible answer:



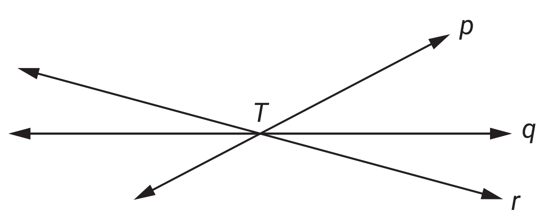

11. Possible answer:




12. Possible answer:

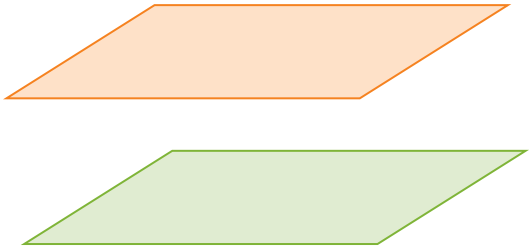



13. Possible answer:



**14.** always; A line contains an infinite number of points, and any of the points can be the endpoint for a pair of opposite rays.

**15.** If the planes are parallel, the planes do not intersect. Possible answer:



**16.** yes; If the three points are colinear, they are contained in an infinite number of planes.

**17.** Possible answer: A utility wire is connected to the side of the building. The point of intersection is the point where the wire touches the building.

**18.** line: $\overleftrightarrow{EF}$; segment: Possible answer: $\overline{AC}$; ray: Possible answer: $\overrightarrow{DB}$

**19.** Postulate 1.3

## Career Preparation: Check

**Test-Taking Tip** Remind students that when questions direct students to "select all," there can be more than one correct answer. They should read each choice carefully to determine whether the choice is a correct answer.

**Answers**

| | | | |
|---|---|---|---|
| **1.** D | **2.** C | **3.** B | **4.** C |
| **5.** a, c, e, f | **6.** a, d, e | **7.** D | **8.** A |
| **9.** B | **10.** C | | |

LESSON 1.5

# Solve Problems Using Pairs of Angles

## CAREER SPOTLIGHT: Occupational Therapist

### Occupation Description

Occupational therapists treat injured, ill, or disabled patients through the therapeutic use of everyday activities. They help these patients develop, recover, improve, and maintain the skills needed for daily living and working.

In some cases, occupational therapists help patients create functional work environments. They evaluate the workspace, recommend modifications, and meet with the patient's employer to collaborate on changes to the patient's work environment or schedule.

### Education

Occupational therapists need at least a master's degree in occupational therapy; some therapists have a doctoral degree. Occupational therapists also must be licensed.

### Potential Employers

The largest employers of occupational therapists are as follows:

| | |
|---|---|
| Hospitals; state, local, and private | 27% |
| Offices of physical, occupational, and speech therapists, and audiologists | 26% |
| Elementary and secondary schools; state, local, and private | 11% |
| Home healthcare services | 9% |
| Nursing care facilities (skilled nursing facilities) | 9% |

**Watch a video** about occupational therapists: https://cdn.careeronestop.org/OccVids/OccupationVideos/29-1122.00.mp4

**Career Cluster**
Health Science

**Career Pathway**
Therapeutic Services

**Career Outlook**
- Salary Projections:
  Low-End Salary, $56,800
  Median Salary, $84,950
  High-End Salary, $121,490
- Jobs in 2018: 133,000
- Job Projections for 2028: 156,800 (increase of 18%)

**Geometry Concepts**
- Apply the Angle Addition Postulate.
- Apply angle bisectors and find complementary and supplementary angles.

**Is this a good career for me?**
Occupational therapists:
- Evaluate a patient's condition and needs
- Develop a treatment plan for patients
- Help people with various disabilities perform different everyday tasks
- Educate a patient's family and employer about how to accommodate and care for the patient

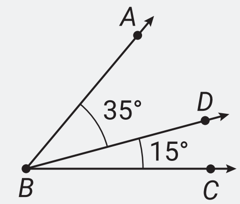
**Lesson Objective**

In this lesson, you will look at how an occupational therapist uses the concepts of angles and angle measurement when working to help patients regain full motion of their arms and legs.

**Angle Addition Postulate**

If $D$ is in the interior of $\angle ABC$, then

$m\angle ABD + m\angle DBC = m\angle ABC$.




35° + 15° = 50°

## 1 Step Into the Career: Angle Measurement

An occupational therapist is working with a patient who has a current range of motion for swinging his right arm up at the shoulder at the angle measure shown in the diagram.

The therapist has a goal of improving the patient's range by 26°. What is the goal for the total angle measure for the patient's range of motion?

An occupational therapist uses a goniometer to measure angles. It is similar to a protractor.

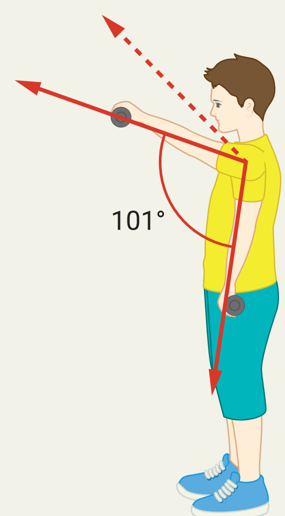



Photo credit: ESB Professional/Shutterstock.com

### Devise a Plan

**Step 1:** Determine the angle measure for the current range of motion.

**Step 2:** Apply the Angle Addition Postulate and write an expression representing the total number of degrees for the goal range of motion.

**Step 3:** Find the value of the expression to determine the total number of degrees.

## Walk Through the Solution

**Step 1:** Determine the current range of motion.

The diagram shows that the current range of motion is 101°.

**Step 2:** Applying the Angle Addition Postulate, the goal for the total range of motion can be found by using the expression 101° + 26°.

**Step 3:** Find the value of the expression.

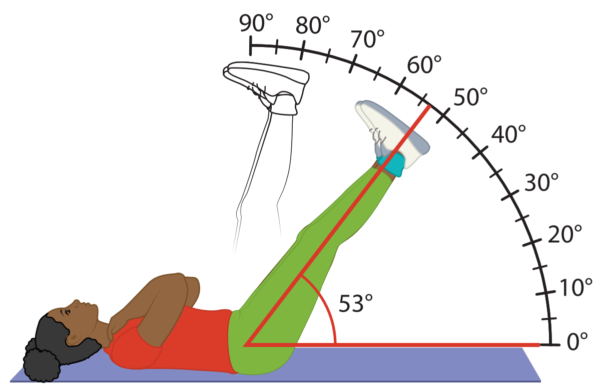
$101° + 26° = 127°$

The goal for the total range of motion is 127°.

## On the Job: Apply Angle Measurement

**1.** Before starting therapy, a patient had a range of motion in her legs of 53°.

After several weeks of therapy, the occupational therapist recorded an angle measure of 77°. The goal for complete recovery is a range of motion of 82°.




**a.** What is the measure of the angle representing the improvement from 53° to 77°?

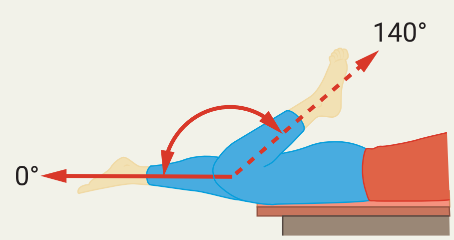
**b.** How many more degrees of motion are needed for complete recovery?

**c.** What is the measure of the angle representing the improvement from 53° to complete recovery at 82°?

## 2 Step Into the Career: Angle Bisector

An occupational therapist is working with a patient who has had major knee surgery and currently has limited motion in her right knee.

The full range of motion for a healthy knee is 140°, as shown in the diagram.




The occupational therapist and patient decide that the first therapy target will be to gain back at least half of the full range of motion. How many degrees represents half of the range of motion?

## Devise a Plan

An **angle bisector** divides an angle into two congruent angles. In this case, the occupational therapist is applying the angle bisector of the angle that represents the full range of motion to determine the first target.

**Step 1:** Determine the full range of motion for a healthy knee.

**Step 2:** Apply the concept of an angle bisector to write an expression representing the measure of each angle resulting from the angle bisector.

**Step 3:** Find the value of the expression to determine the number of degrees that represents half of the full range of motion.

## Walk Through the Solution

**Step 1:** Determine the full range of motion.

The full range of motion for a healthy knee is 140°, according to the information given in the problem.

**Step 2:** The measure of each angle created by the angle bisector can be represented by the expression $\frac{140°}{2}$.

**Step 3:** Find the value of the expression.

$$\frac{140°}{2} = 70°$$

Half of the full range of motion is 70°.

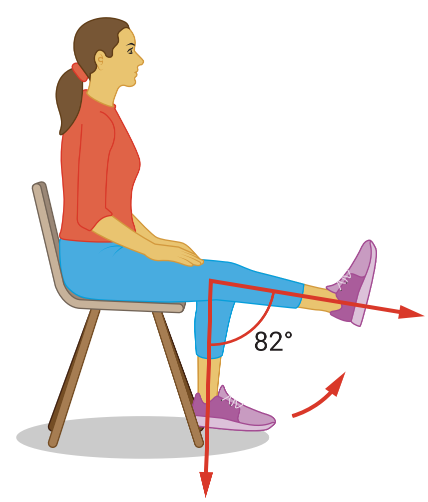
## On the Job: Apply Angle Bisector

2. An occupational therapist is working with a patient doing leg lifts on the right leg. A complete leg lift, bending at the knee, would have the total range of motion angle shown in the diagram.

   The occupational therapist decides that a goal of half the total range of motion is a good first step.

   a. What is the angle measure for the total range of motion?

   b. What is the angle measure for the first step?

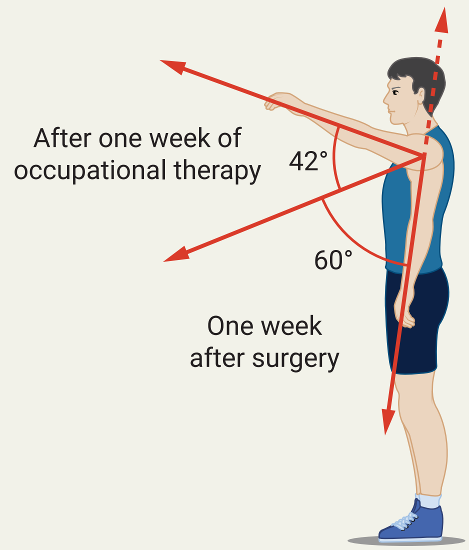


## 3 Step Into the Career: Supplementary or Complementary Angles

A professional baseball player is recovering from left shoulder surgery. The player begins occupational therapy one week after surgery. The occupational therapist's goal is for a return to a full shoulder extension range of motion of 180°, as shown by the dashed arrow.

What is the total range of motion accomplished after one week of occupational therapy?

How many more degrees does the player need to improve to attain the full range of 180°?




### Devise a Plan

**Supplementary angles** are two angles whose measures add to 180°. The goal for the total range total range of motion is 180°. So, the angle for the current amount of progress and the angle still needed for complete motion are supplementary angles.

**Step 1:** Add the range for one week after surgery to the additional range gained after one week of occupational therapy to find the angle of the range of motion for the current progress.

**Step 2:** Since you need to find the angle that is supplementary to the angle for the current progress, subtract the angle for the current progress from 180°.

### Walk Through the Solution

**Step 1:** The range of motion one week after surgery is 60°. The additional range of motion gained after one week of occupational therapy is 42°. Add the angle measures.

$60° + 42° = 102°$

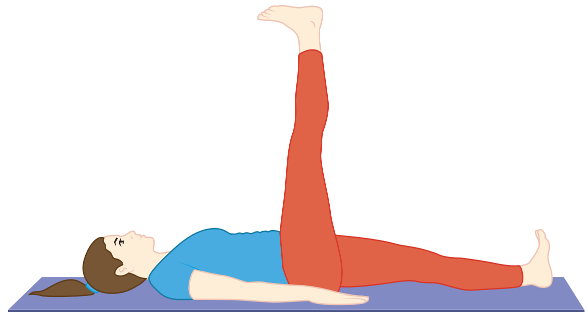
**Step 2:** The full range of motion is 180°. Subtract 102° from 180°.

$180° – 102° = 78°$

To attain full range of motion, the player needs to improve by 78 more degrees.

## On the Job: Apply Supplementary or Complementary Angles

**3.** An occupational therapy patient is lying on a mat working on improving the movement in her right leg. The goal is to move the leg from flat on the floor to extended straight above her. This is a right angle. The current angle she can move her leg to is complementary to the angle she still needs to attain.



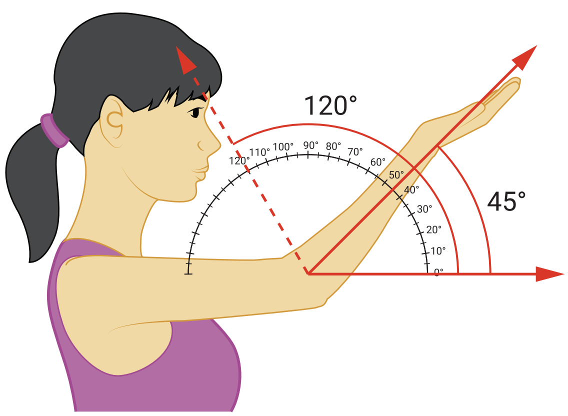
**QUICK TIP**

Complementary angles are two angles whose measures add to 90°.

If the patient can move her leg 28°, how many more degrees does she need to move to reach her goal?

## Career Spotlight: Practice

**4.** After elbow surgery, a patient's range of motion angle for elbow flexion is shown on the goniometer. The dashed arrow indicates the goal range of motion.

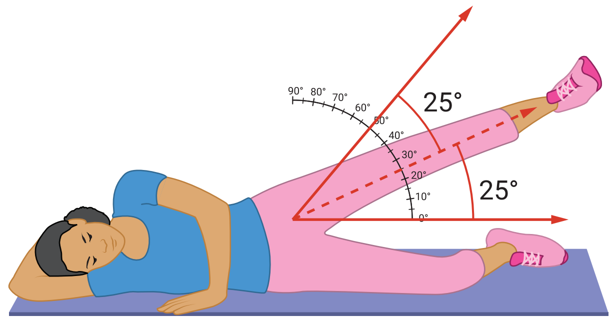



**a.** What is the angle measure for the current range of motion for the patient?

**b.** What is the angle measure for the goal range of motion?

**c.** What is the difference in degrees between the current range of motion and the goal range of motion?

5. Hip abduction is the sideways motion of the leg from the hip, as shown in the picture. The typical full range of motion angle for hip abduction is indicated on the goniometer by the solid arrows. The leg of the person in the picture forms an angle bisector to the angle for the full range of motion, as shown by the dashed arrow.

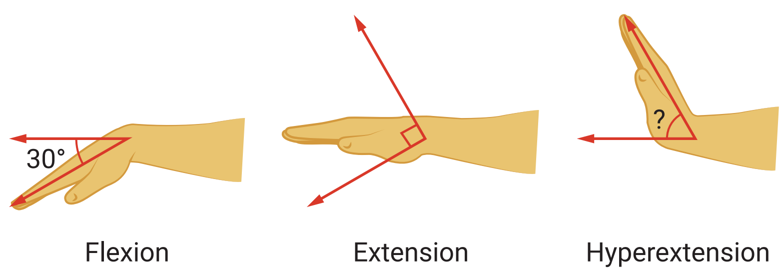



   a. What is the angle measure for the typical full range of motion?

   b. What number of degrees represents half of the typical range of motion?

   c. How many more degrees of motion are necessary for the person to attain the typical range of motion?

6. An occupational therapist uses a goniometer and finds that the right wrist of a patient has a total range of motion from flexion to hyperextension that is a right angle. The range of motion angle from flexion to extension is 30°. What is the measure of the range of motion angle from extension to hyperextension? (**HINT**: The angle is complementary to the angle from flexion to extension.)




## Devise a Plan

**Step 1:** Determine the known angle measures.

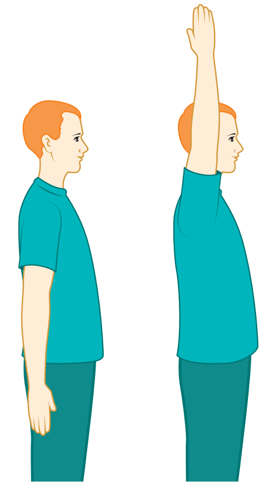
**Step 2:** ______?______.

**Step 3:** ______?______.

7. An occupational therapist is working with a patient on front shoulder extensions for the right arm. This movement begins with the arm hanging loosely at the side. The person keeps the arm straight and lifts the arm in front of the body, finishing with the arm straight upward, as shown in the diagram.

The patient has a 105° range of motion currently. The additional angle the patient needs to be able to move to attain full range of motion is supplementary to the current angle.

What is the measure of the remaining angle?




## Career Spotlight: Check

8. The diagram shows the typical range of motion for a plantar flexion of the ankle.

A gymnast is recovering from ankle surgery and currently has a plantar flexion range of 22°.

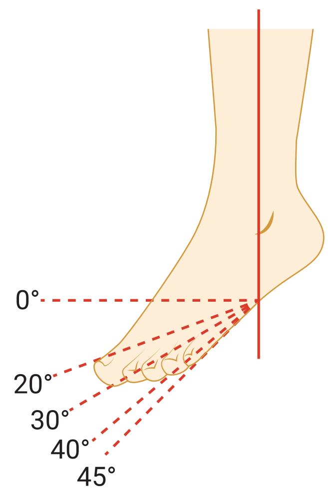
By how many more degrees does the gymnast need to improve to show full range of motion for a plantar flexion?

**A.** 22°

**B.** 23°

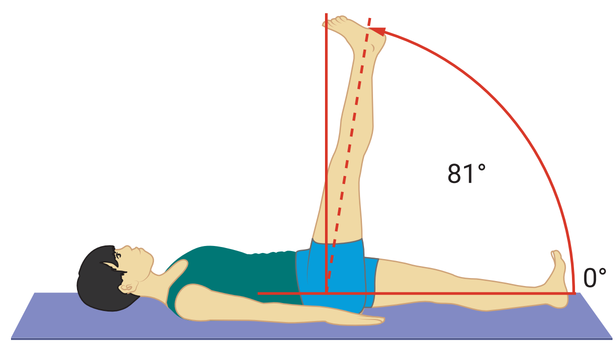
**C.** 45°

**D.** 78°

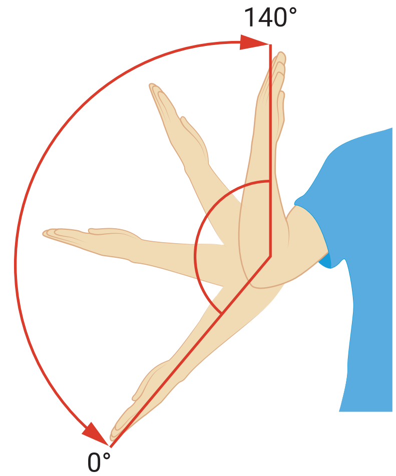



9. An occupational therapist is working with a patient who is recovering from hip surgery.

For the first week of therapy, the goal for the patient's initial range of motion is half of the patient's normal range of motion, which is shown in the diagram. What is the angle measure of the initial goal?



**10.** An occupational therapist is working with a patient who had elbow surgery. The full range of motion for the elbow is 140°, as shown in the diagram. The therapist wants to use 25% of the total range of motion for the first goal and 50% of the total range of motion for the second goal of the therapy. Note that 50% of the full range bisects the full range angle and 25% bisects the 50% angle.




Select the answer from each box that makes the sentence true.

25% of the full range is [ **a.** 25° / **b.** 50° / **c.** 35° ].

50% of the full range is [ **a.** 50° / **b.** 70° / **c.** 140° ].

After attaining 50% of full range, the patient will need another [ **a.** 25° / **b.** 50° / **c.** 70° ] to achieve full range of motion.

**11.** An occupational therapist finds that a patient has a range of motion for a joint of 36°. This is 60% of the typical full range of motion. Which is the typical full range of motion?

**A.** 18°

**B.** 36°

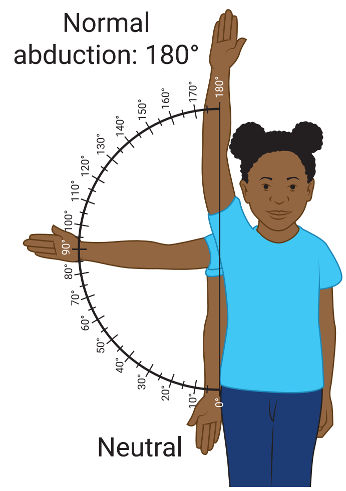
**C.** 45°

**D.** 60°

**12.** An occupational therapist is creating a help sheet for different ranges of motion. She wants to have a quick reference for angles that are 40% of the typical total range of motion angles. Match each total range of motion angle measure with the angle measure representing 40% of the motion.

| | 36° | 48° | 72° |
|---|---|---|---|
| **Wrist flexion to hyperextension, 90°** | ☐ | ☐ | ☐ |
| **Elbow flexion, 120°** | ☐ | ☐ | ☐ |
| **Shoulder neutral to flexion, 180°** | ☐ | ☐ | ☐ |

**13.** The diagram shows range of motion for a vertical abduction of the shoulder.




Select all the statements that are true.

**a.** The angle formed by the angle bisector of the total range of motion is 90°.

**b.** The total range of motion from neutral to abduction is 360°.

**c.** A patient who has a range of 135° of motion for vertical shoulder abduction has 75% of the normal range of motion.

**d.** One-third of the normal range of motion is about 45°.

**e.** Half of the normal range of motion is about 80°.

Teacher Edition

LESSON 1.5

# Solve Problems Using Pairs of Angles

## Common Core State Standards

**G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**Mathematical Practices** 1, 2, 4

## CAREER SPOTLIGHT: Occupational Therapist

Occupational therapists apply science and math to help disabled or injured people recover the ability to perform everyday activities. They work to reestablish and strengthen the physical motion and use of various muscles and limbs in the body. This helps people gain back self-sufficiency in everyday tasks. The techniques used by occupational therapists are drawn from scientific research and involve the math of angles, rotation, and force.

- Discuss occupational therapy with students by reading the Career Spotlight together.
- Find local colleges and universities with occupational therapy programs to share with students.
- Research local hospitals, nursing homes, and clinics that employ occupational therapists and ask about how they utilize occupational therapists.

## Video: Occupational Therapists

Have students watch this video, which describes ways that occupational therapists do their job when working with people.

### Lesson Objective

In this lesson, you will look at how an occupational therapist uses the concepts of angles and angle measurement when working to help patients regain full motion of their limbs.

Photo credit: GagliardiPhotography/Shutterstock.com

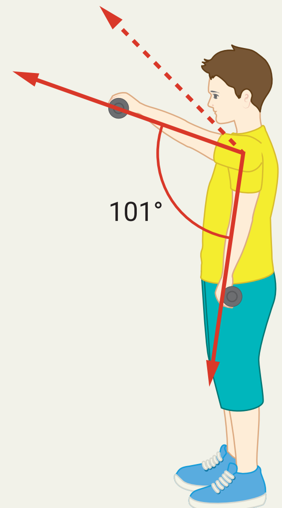
# Teaching Support

## 1 Step Into the Career: Angle Measurement

An occupational therapist is working with a patient who has a current range of motion for swinging his right arm up at the shoulder at the angle measure shown in the diagram.

The therapist has a goal of improving the patient's range by 26°. What is the goal for the total angle measurement for the patient's range of motion?

An occupational therapist uses a goniometer to measure angles. It is similar to a protractor.




If needed, remind students how to measure angles with a protractor.

## Guiding Questions

- In Step 1, what does the diagram tell you about the current range of motion?
- In Step 2, what are the two angle measures that need to be added?
- In Step 3, if the patient wants to increase the range of motion to 180°, how many more degrees does he need to be able to move?

**DIFFERENTIATION: ADDITIONAL SUPPORT** Have a student stand with one shoulder against a wall with a piece of paper taped on. The student should extend the arm forward along the wall, parallel to the floor. Draw a line directly under the arm on the paper. Then have the student bend the arm at the elbow. Draw the angle formed from the elbow to the fingers on the paper. Measure the angle using a protractor. Have the student determine the measure of an angle that is half the first angle and then draw the angle on the paper.

## On the Job: Apply Angle Measurement

**Answers**

**1a.** 24°

**1b.** 5°

**1c.** 29°

**Use these questions to check students' understanding.**

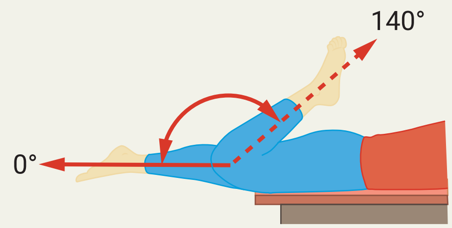
- In 1a, what numbers did you add or subtract to find the answer?
- In 1b, how did you decide what numbers to use to find the answer?
- In 1c, did you need to use 77° to answer the question? Why or why not?

## 2 Step Into the Career: Angle Bisector

An occupational therapist is working with a patient who has had major knee surgery and currently has limited motion in her right knee.

The full range of motion for a healthy knee is 140°, as shown in the diagram.

The occupational therapist and patient decide that the first therapy target will be to gain back at least half of the full range of motion. How many degrees represents half of the range of motion?




### Guiding Questions

- In Step 2, how do you know to divide by 2?
- What is the measure of the angles formed by the bisector of a 70° angle?

**LANGUAGE SUPPORT** Some students may have difficulty with the concepts of *range of motion* and *angle bisector*. Have students use a protractor to measure the angle formed when making the widest V shape with the first two fingers of one hand.

- Tell students that the range of motion refers to the possible amount of movement from the fingers being together to the fingers being in the V shape. Have students draw the angle representing their range of motion for the V shape.
- Tell students that the prefix *bi-* means *two*. For example, a bicycle has two wheels. An angle bisector divides an angle into two equal pieces or sections. Have students use a protractor to draw the angle bisector for their angles.

## On the Job: Apply Angle Bisector

**Answers**

**2a.** 82°

**2b.** 41°

**Use these questions to check students' understanding.**

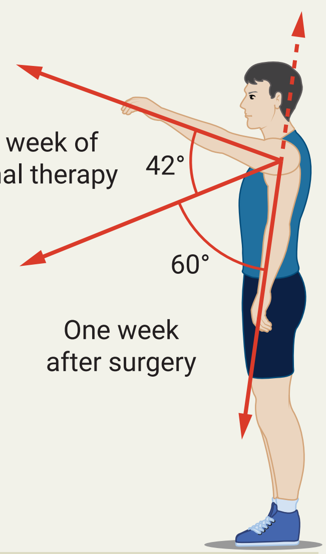
- In 2a, how did you determine the total range of motion?
- In 2b, what expression did you use to find the answer?

## 3 Step Into the Career: Supplementary or Complementary Angles

A professional baseball player is recovering from left shoulder surgery. The player begins occupational therapy one week after surgery. The occupational therapist's goal is for a return to a full shoulder extension range of motion of 180°, as shown by the dashed arrow.

What is the total range of motion accomplished after one week of occupational therapy?

How many more degrees does the player need to improve to attain the full range of 180°?




Have students recall the definitions of supplementary and complementary angles.

## Guiding Questions

- In Step 1, what two numbers do you need to add?
- In Step 2, what are the two supplementary angles? How can you check that they are supplementary?
- If the player wanted to extend his full range of motion by 12° more, what would the full range of motion be?

**EXTENSION** Suppose a patient is working on developing a full range of motion in a limb of 120°. The patient is starting with only 5° of motion. The occupational therapist makes the following goals for percent of total range of motion for the end of each week of occupational therapy.

| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 |
|---|---|---|---|---|---|
| 25% | 50% | 60% | 75% | 90% | 100% |

What is the total number of degrees of motion that need to be accomplished each week?
(Answer: Week 1: 30°, Week 2: 60°, Week 3: 72°, Week 4: 90°, Week 5: 108°, Week 6: 120°)

## On the Job: Apply Supplementary or Complementary Angles

**Answer**

**3.** 62°

**Use these questions to check students' understanding.**

- What angle is complementary to 28°?
- What numbers did you subtract to find the answer?
- If the full range of motion goal is 140° instead of 90°, how many more degrees would be necessary to reach the goal?

## Career Spotlight: Practice

**Solution Steps for Exercises 4–7**

These steps will help guide students in solving these practice exercises.

### Exercise 4

**Answers**

**4a.** 45°

**4b.** 120°

**4c.** 75°

**Solution Steps**

- Read the measure on the goniometer (protractor) for the first angle. (45°)
- Read the measure on the goniometer (protractor) for the goal range of motion angle. (120°)
- Subtract 45° from 120° to find the difference in the two ranges. (75°)

### Exercise 5

**Answers**

**5a.** 50°

**5b.** 25°

**5c.** 25°

**Solution Steps**

- Read the measure on the goniometer (protractor) for the full range of motion. (50°)
- Divide the full range of motion, 50°, by 2 to find half the range. (25°)
- Since the angle is bisected, the angle measure that the patient requires is the same as half the range of motion. (25°)

## Exercise 6

**Answer**

**6.** 60°

**Devise a Plan**

Possible plan:

**Step 1:** Determine the known angle measures.

**Step 2:** Write an expression to find an angle complementary to 30°.

**Step 3:** Subtract to find the angle.

**Solution Steps**

- The known angles are 30° and 90°. The first angle is complementary to the unknown angle.
- Write an express to find an angle complementary to 30°. (90° – 30°)
- Subtract to find the angle. (60°)

## Exercise 7

**Answer**

**7.** 75°

**Solution Steps**

- Understand that the full range of motion forms a straight angle, which is 180°.
- Subtract 105° from 180° to find the remaining angle. (75°)

## Career Spotlight: Check

**Tips for Completing Exercises 8–13**

These tips will help students in solving these exercises and similar assessment items.

## Exercise 8

**Answer**

**8.** B

**Tip** Encourage students to study the diagram to determine the full range of motion. Remind students that to find the difference between two numbers, they should subtract.

## Exercise 9

**Answer**

**9.** 40.5°

**Tip** Encourage students to read the problem carefully and observe the measurement shown on the diagram. The range of motion is shown in the diagram. Ask students to think about how to find a number that when multiplied by two gives the angle measure for the range of motion.

## Exercise 10

**Answer**

**10.** c. 35°, b. 70°, c. 70°

**Tip** Encourage students to check their answer for reasonableness after completing the problem. For example, the answers for the second and third boxes both represent 50% of the full range of motion. They should be the same number.

## Exercise 11

**Answer**

**11.** D

**Tip** Encourage students to read the problem carefully. The range given is less than 100% of the full range of motion. A number representing 100% of the full range will be larger than the given number. Based on this, students can quickly eliminate two of the answer choices and then concentrate on finding the actual number.

## Exercise 12

**Answer**

**12.** Wrist flexion to hyperextension, 90°: 36°, Elbow flexion, 120°: 48°, Shoulder neutral to flexion, 180°: 72°

**Tip** Remind students that each range matches only one choice. If it helps, students can draw the angles using a protractor to visualize what 40% of each measure could look like.

## Exercise 13

**Answer**

**13.** a, c

**Tip** Encourage students to study the diagram and then analyze each statement according to what is shown in the diagram. For example, the full range of motion is given in the diagram. All the calculations that confirm or refute each statement can be made based on this information.

# Notes